

大规模三维重建中的增量式运动平均问题

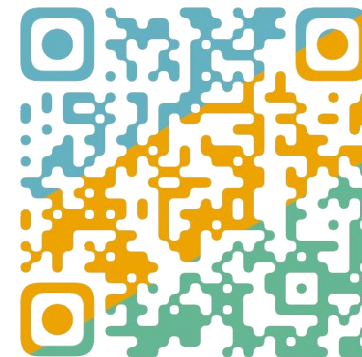
Incremental Motion Averaging in Large-Scale 3D Reconstruction

高翔

中国科学院自动化研究所

电子邮箱: xiang.gao@ia.ac.cn

个人主页: <https://xgao-3dv.github.io/>



研究背景

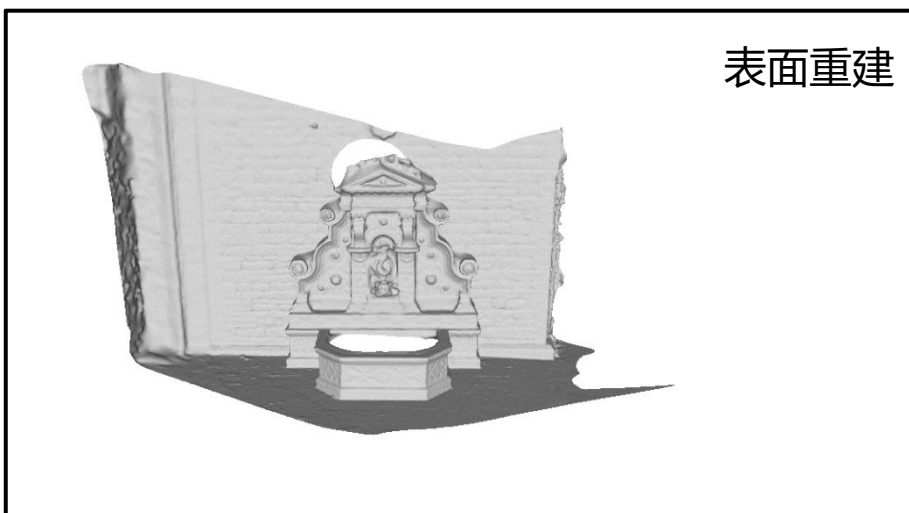
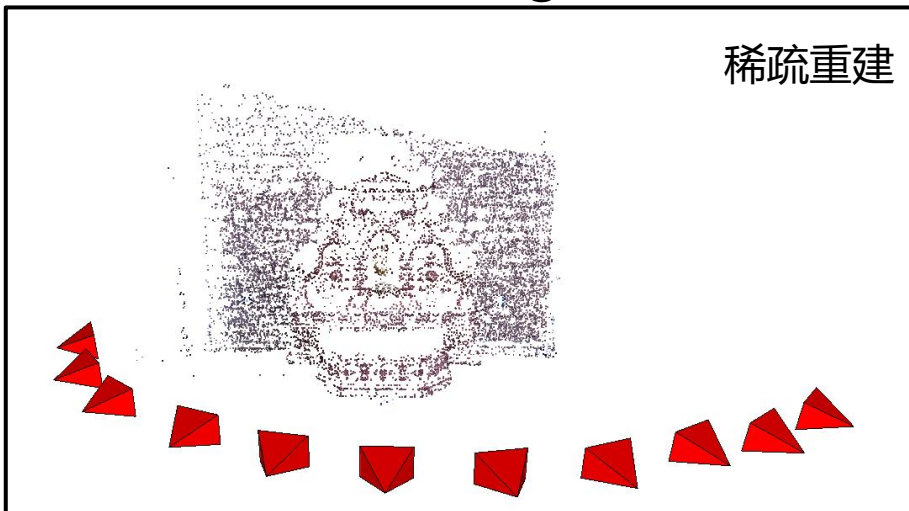
- 从运动恢复结构 (Structure from Motion, SfM)
 - SfM: 输入图像集合, 输出**相机绝对位姿与场景稀疏表达**



Snaveley et al. PIEEE 2010

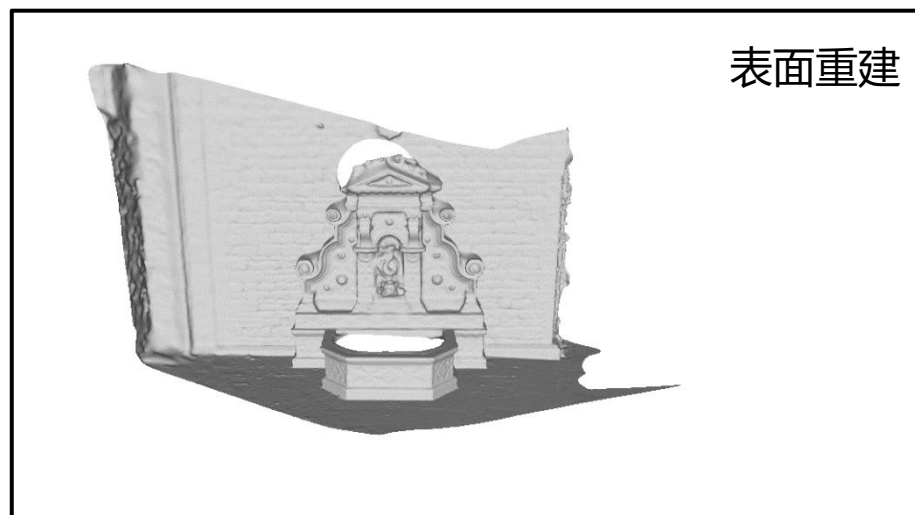
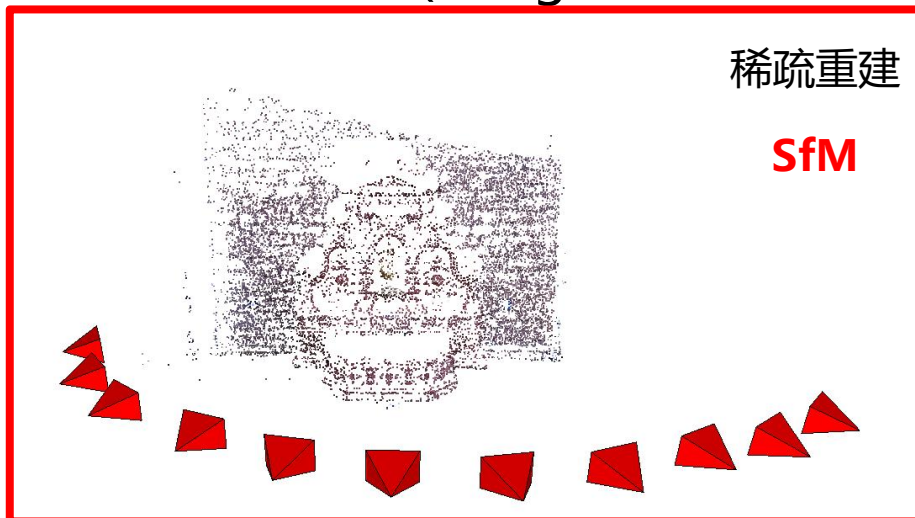
研究背景

- 从运动恢复结构 (Structure from Motion, SfM)
 - SfM是基于图像的建模 (Image-Based Modeling) 流程中的核心步骤



研究背景

- 从运动恢复结构 (Structure from Motion, SfM)
 - SfM是基于图像的建模 (Image-Based Modeling) 流程中的核心步骤

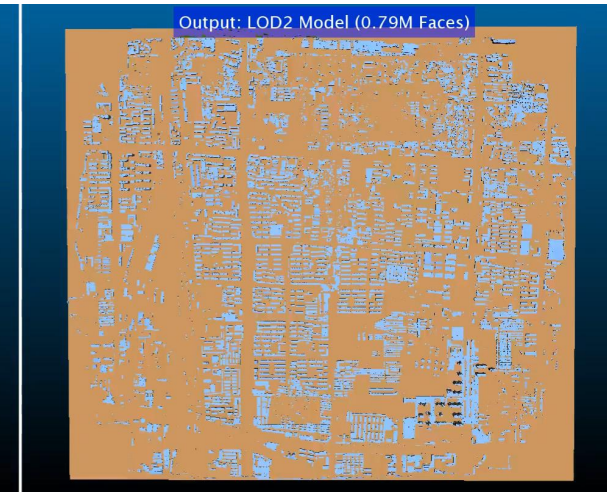
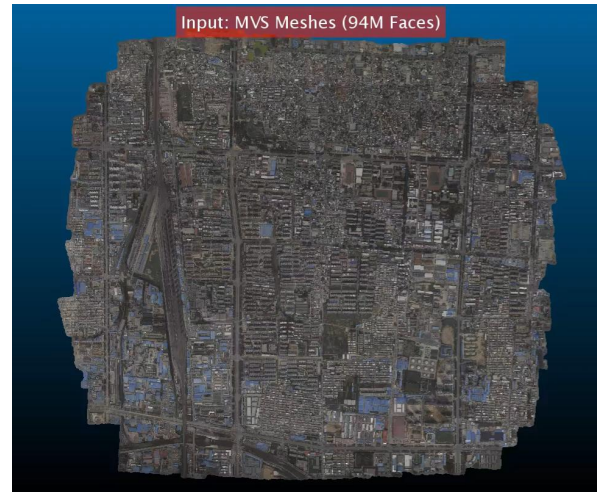


研究背景

- 从运动恢复结构 (Structure from Motion, SfM)
 - SfM在**室内建模**、遥感测绘、古建保护、增强现实等领域有着重要的应用



Han et al. ISPRS P&RS 2021



Zhu et al. In Proc. ECCV 2018



Gao et al. ISPRS P&RS 2018



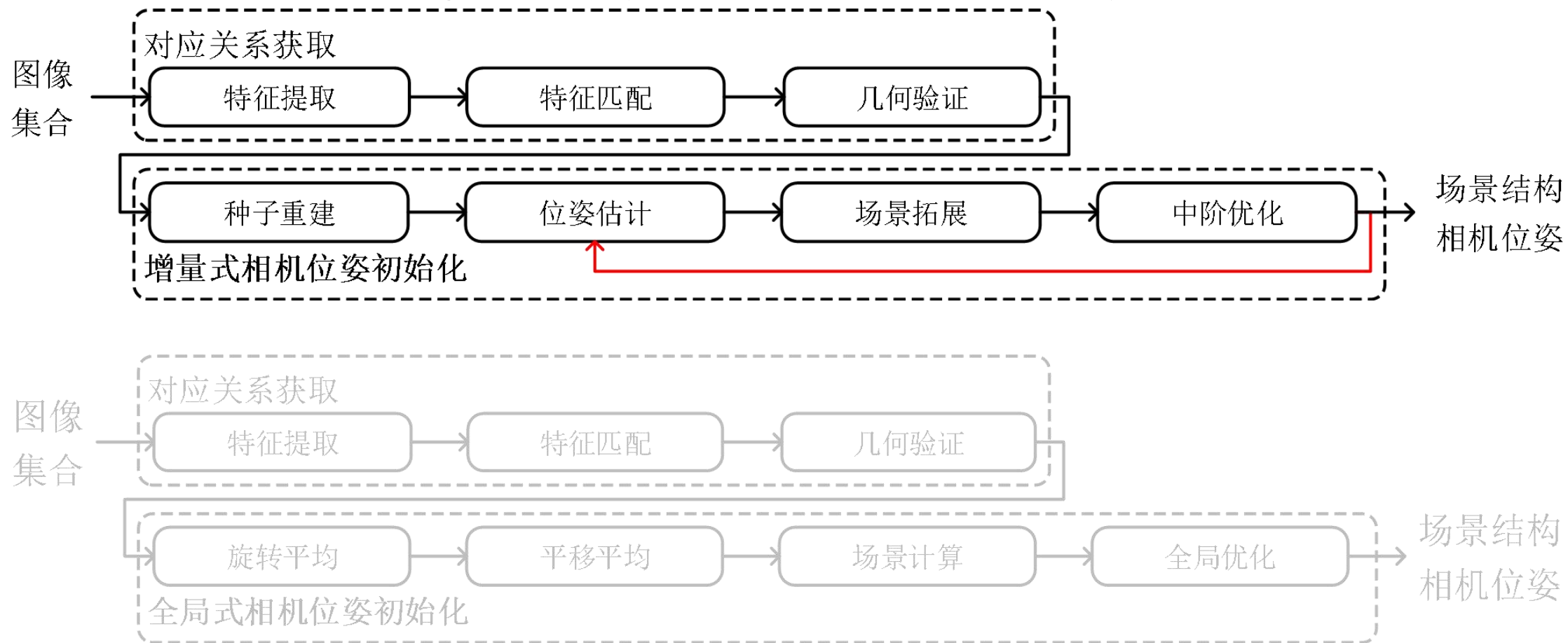
Sun et al. In Proc. CVPR 2021



研究背景

- 从运动恢复结构 (Structure from Motion, SfM)

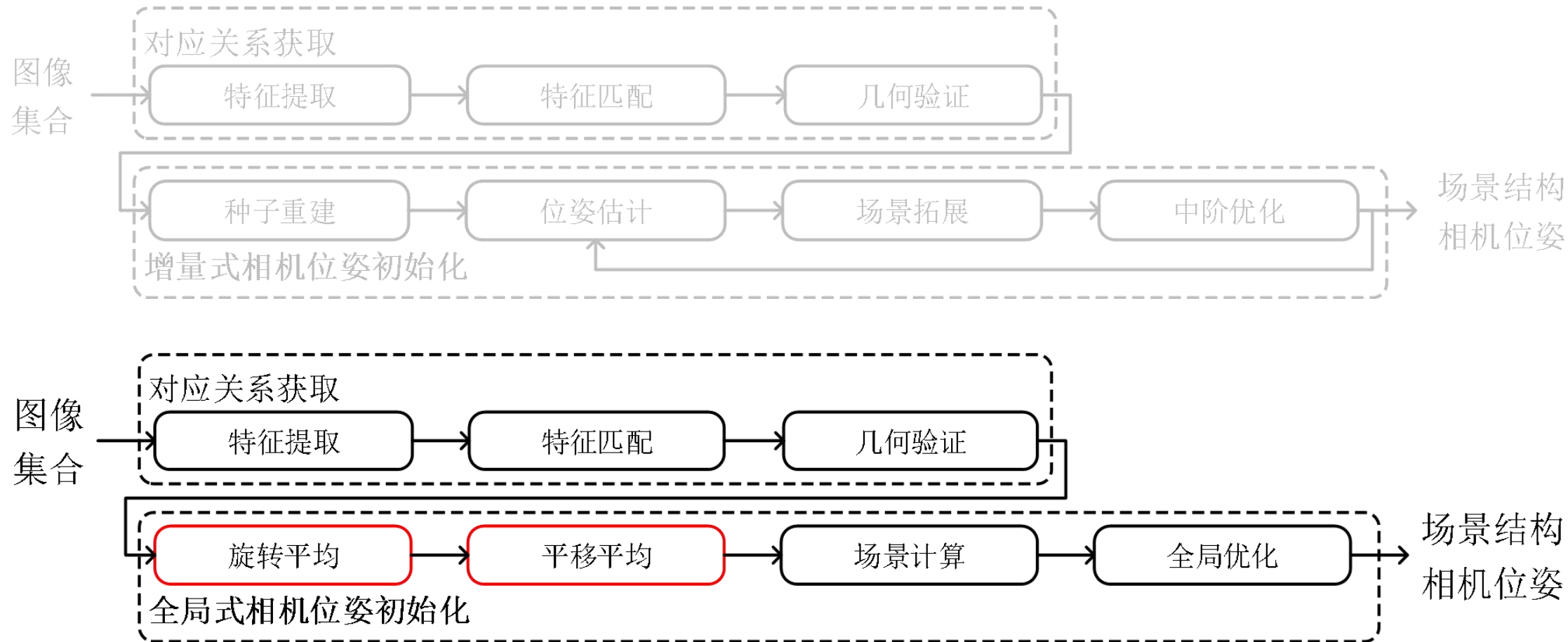
- 根据相机位姿初始化方式, SfM可分为**增量式SfM**与**全局式SfM**两类



研究背景

- 从运动恢复结构 (Structure from Motion, SfM)

- 根据相机位姿初始化方式, SfM可分为增量式SfM与**全局式SfM**两类



研究背景

- 增量式SfM vs. 全局式SfM
 - 增量式SfM
 - ◆ 精度更高、鲁棒性更强、
 - ◆ 效率较低、场景漂移、
 - 全局式SfM
 - ◆ 效率更高、一致性更好、
 - ◆ 精度较差、场景缺失、

研究背景

- 运动平均 (Motion Averaging)

- 运动平均包括旋转平均 (Rotation Averaging, RA) 与平移平均 (Translation Averaging, TA) , 是**全局式**SfM中的核心步骤

- RA: 给定相机之间的相对旋转 $\{\mathbf{R}_{i,j} | e_{i,j} \in \mathcal{E}\}$, 求解在同一基准下所有相机的绝对旋转 $\{\mathbf{R}_i | v_i \in \mathcal{V}\}$

$$\{\mathbf{R}_i^*\} = \arg \min \sum_{\substack{e_{i,j} \in \mathcal{E} \\ v_i, v_j \in \mathcal{V}}} \rho \left(d_R(\mathbf{R}_{i,j}, \mathbf{R}_j \mathbf{R}_i^\top) \right)$$

研究背景

- 运动平均 (Motion Averaging)

- 运动平均包括旋转平均 (Rotation Averaging, RA) 与平移平均 (Translation Averaging, TA) , 是**全局式**SfM中的核心步骤

- RA: 给定相机之间的相对旋转 $\{\mathbf{R}_{i,j} | e_{i,j} \in \mathcal{E}\}$, 求解在同一基准下所有相机的绝对旋转 $\{\mathbf{R}_i | v_i \in \mathcal{V}\}$

$$\{\mathbf{R}_i^*\} = \arg \min_{\substack{e_{i,j} \in \mathcal{E} \\ v_i, v_j \in \mathcal{V}}} \sum \rho \left(d_R(\mathbf{R}_{i,j}, \mathbf{R}_j \mathbf{R}_i^\top) \right)$$

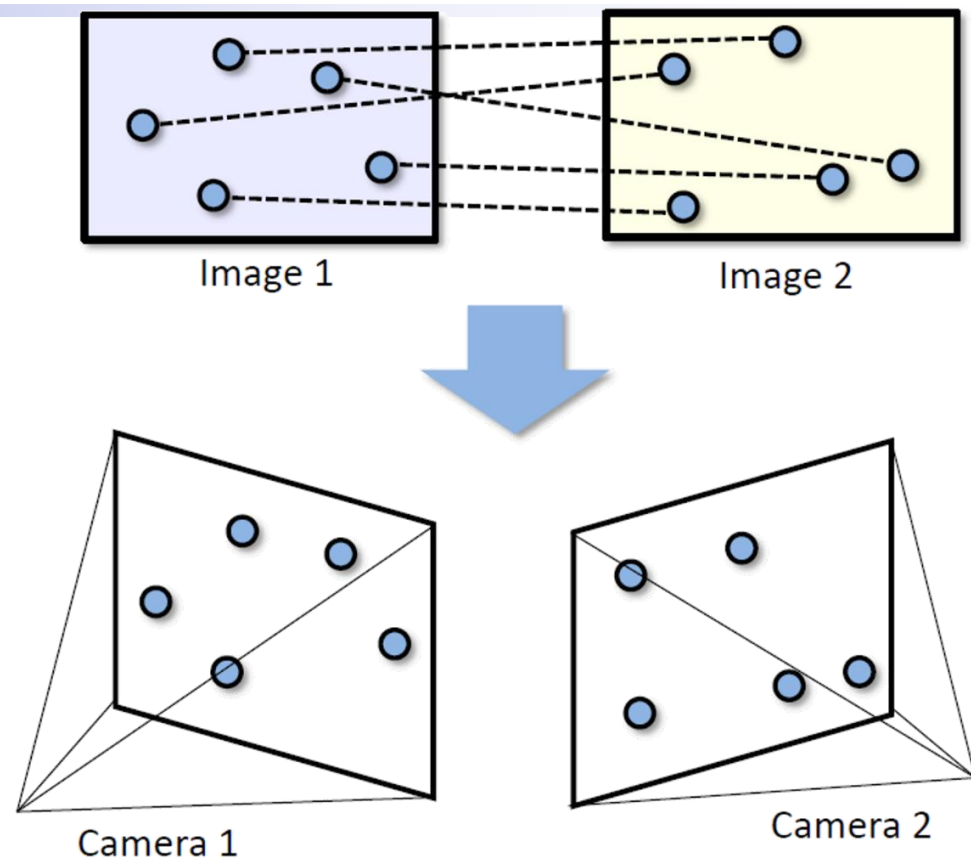
- TA: 给定相机之间的相对平移 $\{\mathbf{t}_{i,j} | e_{i,j} \in \mathcal{E}\}$, 求解在同一基准下所有相机的绝对位置 $\{\mathbf{c}_i | v_i \in \mathcal{V}\}$

$$\{\mathbf{c}_i^*\} = \arg \min_{\substack{e_{i,j} \in \mathcal{E} \\ v_i, v_j \in \mathcal{V}}} \sum \rho \left(d_t \left(\mathbf{t}_{i,j}, \mathbf{R}_j \frac{\mathbf{c}_i - \mathbf{c}_j}{\|\mathbf{c}_i - \mathbf{c}_j\|_2} \right) \right)$$

研究背景

- 运动平均 (Motion Averaging, MA)

- 相对运动 (旋转、平移) $\{R_{i,j}, t_{i,j} | e_{i,j} \in \mathcal{E}\}$ 的获取方式:
 - 局部特征的**提取与匹配**
 - 本质矩阵的**估计与分解**



研究背景

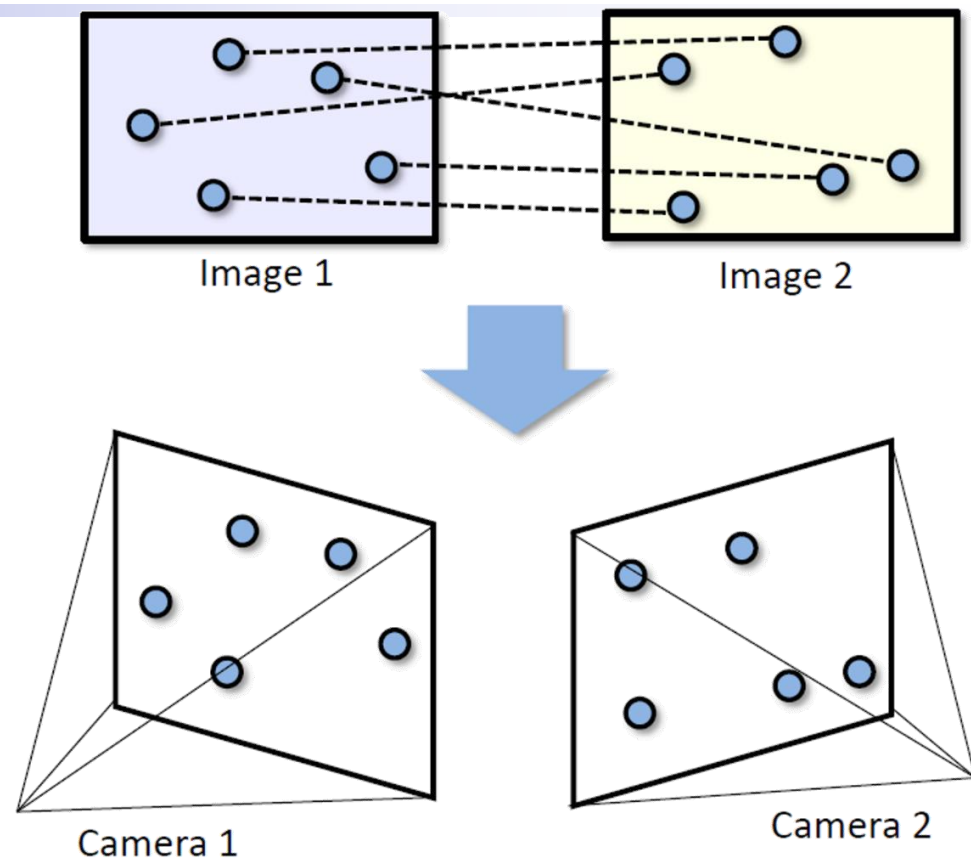
- 运动平均 (Motion Averaging, MA)

- 相对运动 (旋转、平移) $\{R_{i,j}, t_{i,j} | e_{i,j} \in \mathcal{E}\}$ 的获取方式:

- 局部特征的提取与匹配
- 本质矩阵的估计与分解

- 相对于RA, TA问题更加困难, 主要体现在

- 通过本质矩阵分解得到的相对平移具有**尺度不确定性**^[1]
- 相对平移估计精度比旋转更容易受到**特征误匹配**影响^[1]
- 平移平均的**问题可解性**对外极几何图有着更高的要求^[2]



[1] D. Nister. An Efficient Solution to the Five-Point Relative Pose Problem[J]. *IEEE T-PAMI*, 2004.

[2] O. Ozyesil and A. Singer. Robust Camera Location Estimation by Convex[C]. *In Proc. CVPR*, 2015.

研究背景

- 运动平均 (Motion Averaging)

- 运动平均问题的**主要难点**在于如何利用存在测量外值的相对运动精确估计相机的绝对位姿
- 现有的运动平均方法主要包括两类：
 - 鲁棒优化方法：RA^[1-2]、TA^[3-4]，形式复杂、效率欠佳
 - 外值滤除方法：RA^[5-6]、TA^[7-8]，chicken-and-egg problem

[1] A. Chatterjee and V. M. Govindu. Robust Relative Rotation Averaging[J]. *IEEE T-PAMI*, 2018.

[2] Y. Shi and G. Lerman. Message Passing Least Squares Framework and its Application to Rotation Synchronization[C]. *In Proc. ICML*, 2020.

[3] T. Goldstein, P. Hand, C. Lee, V. Voroninski, and S. Soatto. ShapeFit and ShapeKick for Robust, Scalable Structure from Motion[C]. *In Proc. ECCV*, 2016.

[4] B. Zhuang, L. Cheong, and G. H. Lee. Baseline Desensitizing in Translation Averaging[C]. *In Proc. CVPR*, 2018.

[5] X. Gao, J. Luo, K. Li, and Z. Xie. Hierarchical RANSAC-Based Rotation Averaging[J]. *IEEE SPL*, 2020.

[6] S. H. Lee and J. Civera. HARA: A Hierarchical Approach for Robust Rotation Averaging[C]. *In Proc. CVPR*, 2022.

[7] K. Wilson and N. Snavely. Robust Global Translations with 1DSfM[C]. *In Proc. ECCV*, 2014.

[8] C. Sweeney, T. Sattler, T. Höllerer T, M. Turk, and M. Pollefeys. Optimizing the Viewing Graph for Structure-from-Motion[C]. *In Proc. ICCV*, 2015.

研究背景

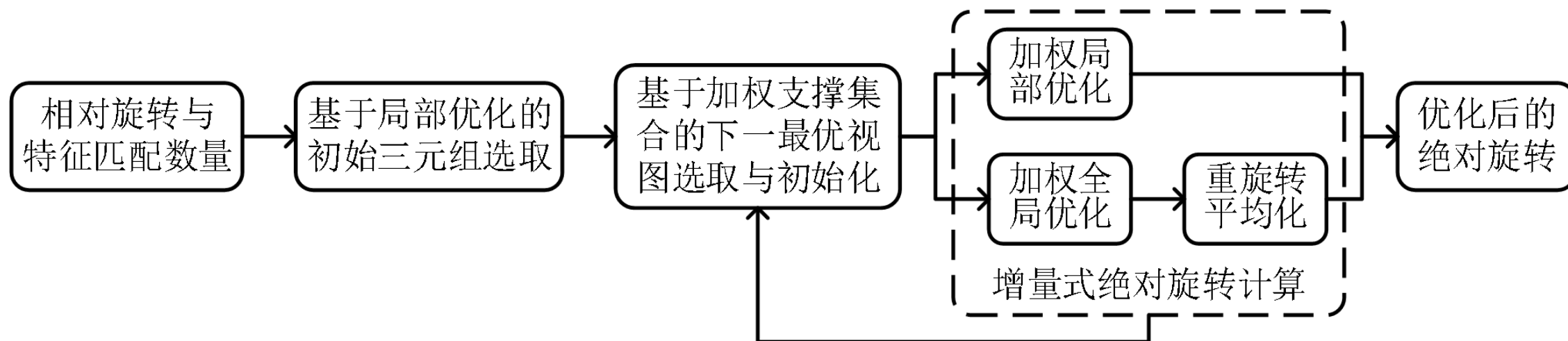
- 运动平均 (Motion Averaging)

- 运动平均问题的**主要难点**在于如何利用存在测量外值的相对运动精确估计相机的绝对位姿
- 现有的运动平均方法主要包括两类：
 - 鲁棒优化方法: RA^[1-2]、TA^[3-4], 形式复杂、效率欠佳
 - 外值滤除方法: RA^[5-6]、TA^[7-8], chicken-and-egg problem
- 针对上述问题, 我们在大规模运动平均的鲁棒性问题研究方面开展了一些初步探索:
 - 可在滤除相对运动测量外值**同时**估计相机的绝对位姿
 - 以期实现简单、高效、精确、鲁棒的**大规模**运动平均

旋转平均 (Rotation Averaging, RA)

• 增量式旋转平均 (Incremental Rotation Averaging, IRA)

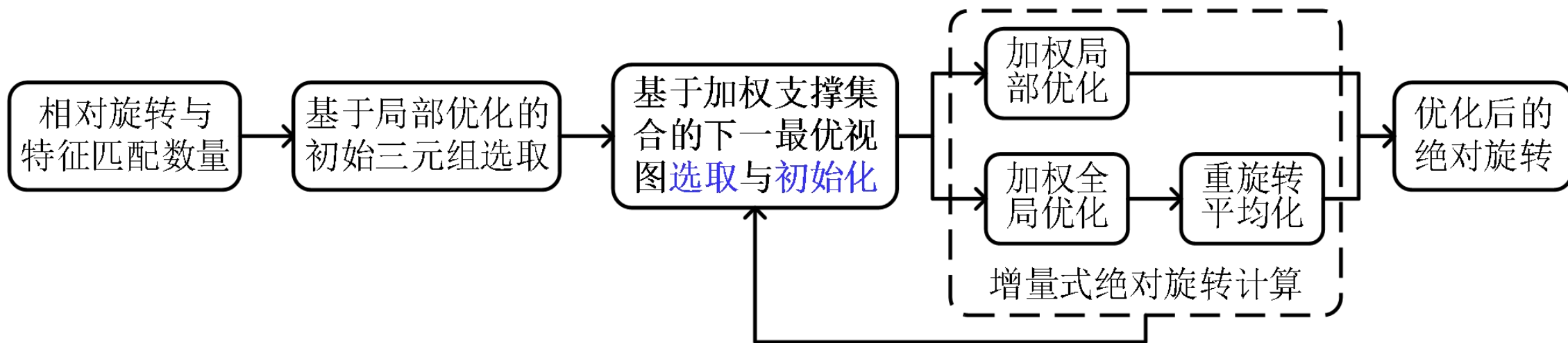
- IRA采用与增量式SfM类似的**增量式**参数估计流程
 - 更加精确、鲁棒
- 旋转平均问题相对于SfM问题待估计的**参数量**更少
 - 更加简单、高效



旋转平均 (Rotation Averaging, RA)

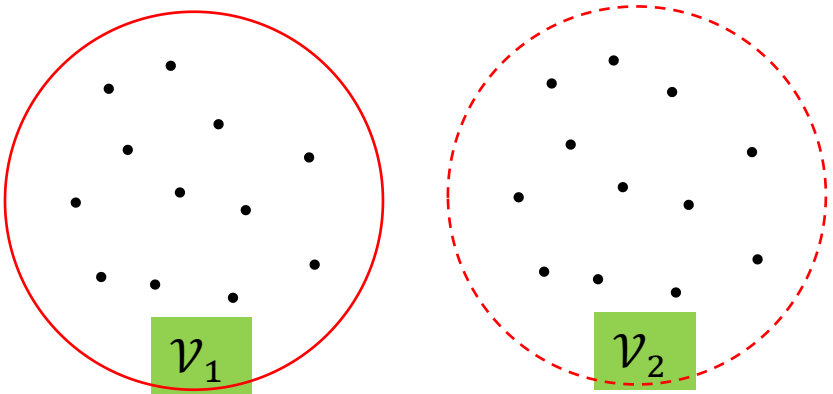
• 增量式旋转平均 (Incremental Rotation Averaging, IRA)

- IRA采用与增量式SfM类似的**增量式**参数估计流程
 - 更加精确、鲁棒
- 旋转平均问题相对于SfM问题待估计的**参数量**更少
 - 更加简单、高效



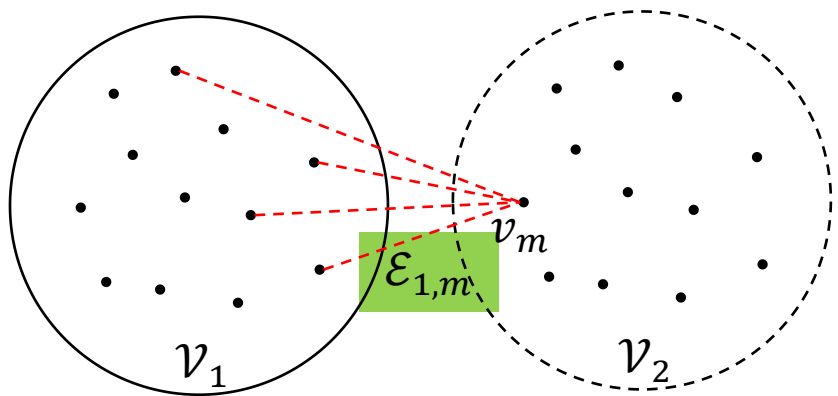
旋转平均 (Rotation Averaging, RA)

- IRA下一最优视图 (Next Best View, NBV) 选取与初始化
 - \mathcal{V}_1 与 \mathcal{V}_2 : 当前已估计与未估计绝对旋转的顶点集合



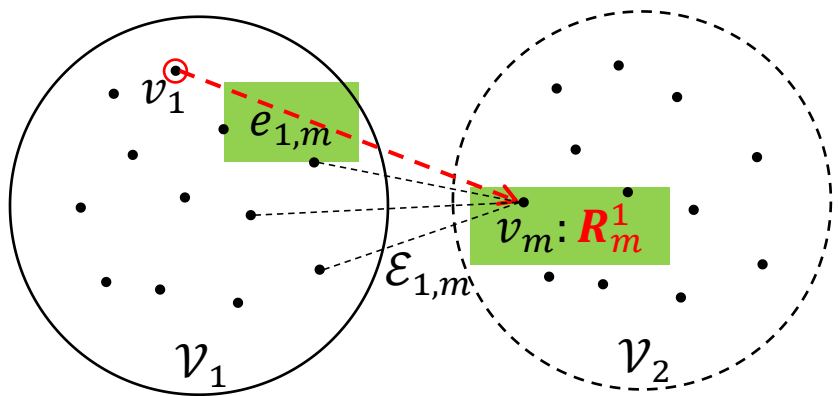
旋转平均 (Rotation Averaging, RA)

- IRA下一最优视图 (Next Best View, NBV) 选取与初始化
 - \mathcal{V}_1 与 \mathcal{V}_2 : 当前已估计与未估计绝对旋转的顶点集合
 - $\mathcal{E}_{1,m}$: \mathcal{V}_2 中一顶点 v_m 与 \mathcal{V}_1 中所有顶点之间的边集



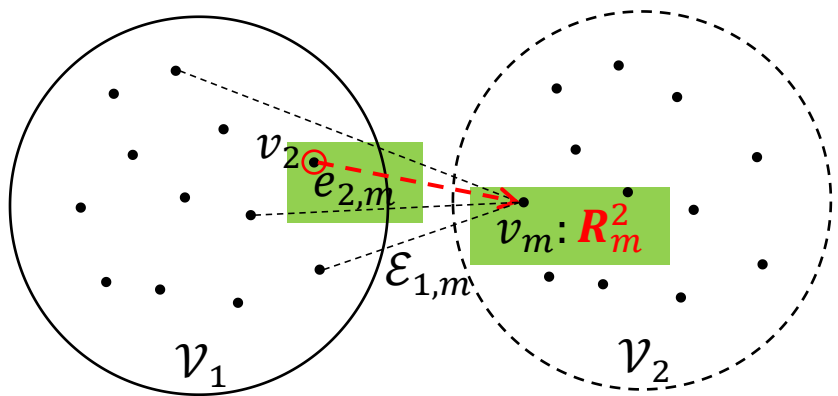
旋转平均 (Rotation Averaging, RA)

- IRA下一最优视图 (Next Best View, NBV) 选取与初始化
 - \mathcal{V}_1 与 \mathcal{V}_2 : 当前已估计与未估计绝对旋转的顶点集合
 - $\mathcal{E}_{1,m}$: \mathcal{V}_2 中一顶点 v_m 与 \mathcal{V}_1 中所有顶点之间的边集
 - 对 $\mathcal{E}_{1,m}$ 中的每一条边 $e_{i,m}$ 预计算顶点 v_m 的绝对旋转 R_m^i



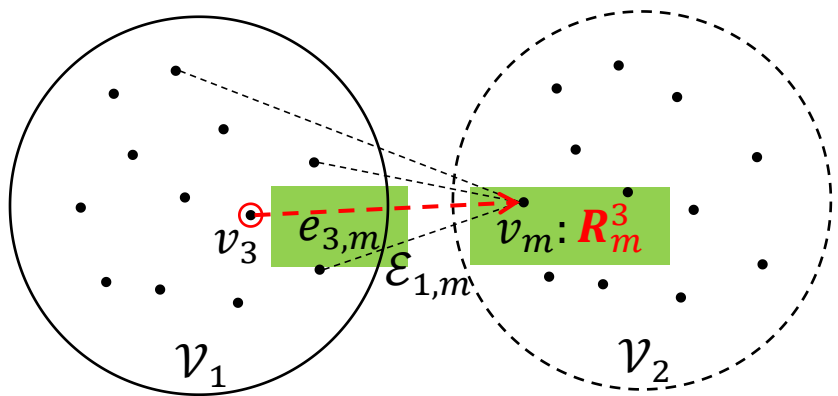
旋转平均 (Rotation Averaging, RA)

- IRA下一最优视图 (Next Best View, NBV) 选取与初始化
 - \mathcal{V}_1 与 \mathcal{V}_2 : 当前已估计与未估计绝对旋转的顶点集合
 - $\mathcal{E}_{1,m}$: \mathcal{V}_2 中一顶点 v_m 与 \mathcal{V}_1 中所有顶点之间的边集
 - 对 $\mathcal{E}_{1,m}$ 中的每一条边 $e_{i,m}$ 预计算顶点 v_m 的绝对旋转 R_m^i



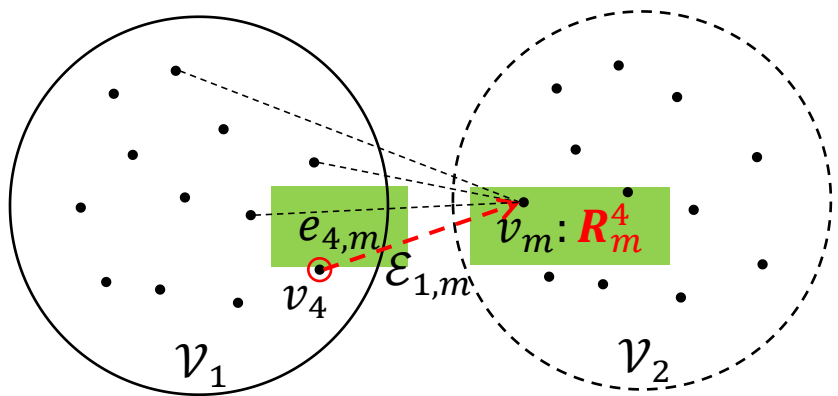
旋转平均 (Rotation Averaging, RA)

- IRA下一最优视图 (Next Best View, NBV) 选取与初始化
 - \mathcal{V}_1 与 \mathcal{V}_2 : 当前已估计与未估计绝对旋转的顶点集合
 - $\mathcal{E}_{1,m}$: \mathcal{V}_2 中一顶点 v_m 与 \mathcal{V}_1 中所有顶点之间的边集
 - 对 $\mathcal{E}_{1,m}$ 中的每一条边 $e_{i,m}$ 预计算顶点 v_m 的绝对旋转 R_m^i



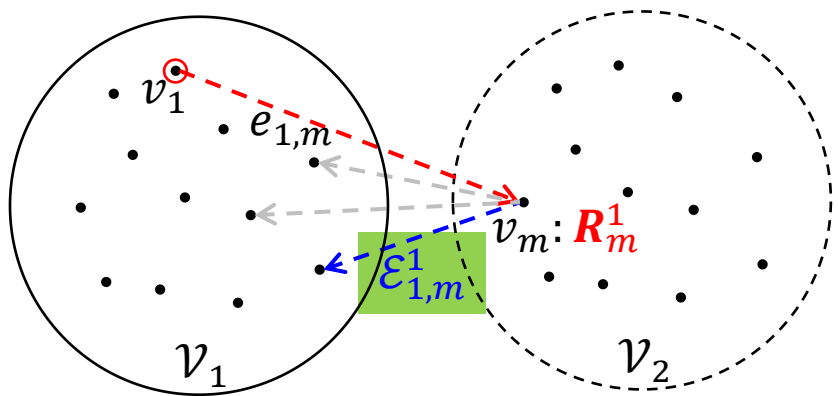
旋转平均 (Rotation Averaging, RA)

- IRA下一最优视图 (Next Best View, NBV) 选取与初始化
 - \mathcal{V}_1 与 \mathcal{V}_2 : 当前已估计与未估计绝对旋转的顶点集合
 - $\mathcal{E}_{1,m}$: \mathcal{V}_2 中一顶点 v_m 与 \mathcal{V}_1 中所有顶点之间的边集
 - 对 $\mathcal{E}_{1,m}$ 中的每一条边 $e_{i,m}$ 预计算顶点 v_m 的绝对旋转 R_m^i



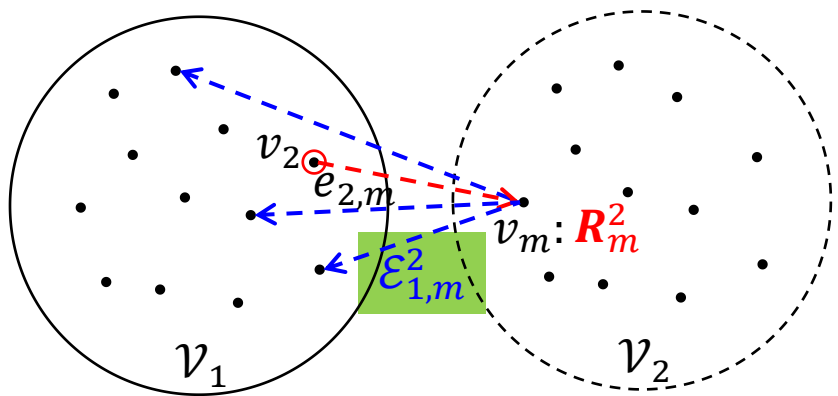
旋转平均 (Rotation Averaging, RA)

- IRA下一最优视图 (Next Best View, NBV) 选取与初始化
 - \mathcal{V}_1 与 \mathcal{V}_2 : 当前已估计与未估计绝对旋转的顶点集合
 - $\mathcal{E}_{1,m}$: \mathcal{V}_2 中一顶点 v_m 与 \mathcal{V}_1 中所有顶点之间的边集
 - 对 $\mathcal{E}_{1,m}$ 中的每一条边 $e_{i,m}$ 预计算顶点 v_m 的绝对旋转 R_m^i
 - 对 v_m 的每一个预计算绝对旋转 R_m^i 获取支撑边集 $\mathcal{E}_{1,m}^i$



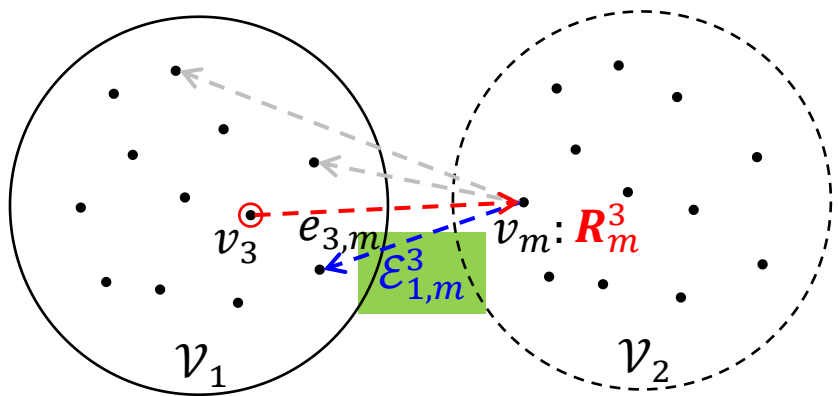
旋转平均 (Rotation Averaging, RA)

- IRA下一最优视图 (Next Best View, NBV) 选取与初始化
 - \mathcal{V}_1 与 \mathcal{V}_2 : 当前已估计与未估计绝对旋转的顶点集合
 - $\mathcal{E}_{1,m}$: \mathcal{V}_2 中一顶点 v_m 与 \mathcal{V}_1 中所有顶点之间的边集
 - 对 $\mathcal{E}_{1,m}$ 中的每一条边 $e_{i,m}$ 预计算顶点 v_m 的绝对旋转 R_m^i
 - 对 v_m 的每一个预计算绝对旋转 R_m^i 获取支撑边集 $\mathcal{E}_{1,m}^i$



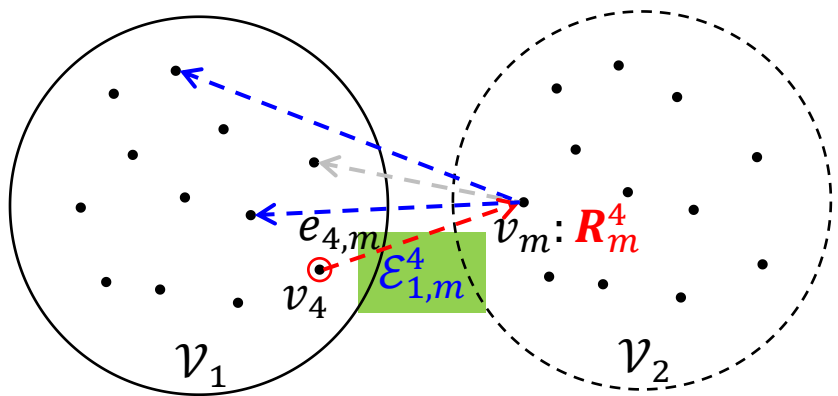
旋转平均 (Rotation Averaging, RA)

- IRA下一最优视图 (Next Best View, NBV) 选取与初始化
 - \mathcal{V}_1 与 \mathcal{V}_2 : 当前已估计与未估计绝对旋转的顶点集合
 - $\mathcal{E}_{1,m}$: \mathcal{V}_2 中一顶点 v_m 与 \mathcal{V}_1 中所有顶点之间的边集
 - 对 $\mathcal{E}_{1,m}$ 中的每一条边 $e_{i,m}$ 预计算顶点 v_m 的绝对旋转 R_m^i
 - 对 v_m 的每一个预计算绝对旋转 R_m^i 获取支撑边集 $\mathcal{E}_{1,m}^i$



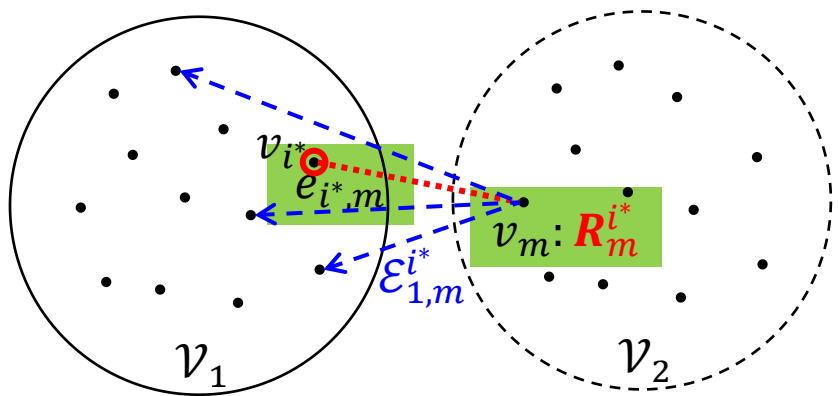
旋转平均 (Rotation Averaging, RA)

- IRA下一最优视图 (Next Best View, NBV) 选取与初始化
 - \mathcal{V}_1 与 \mathcal{V}_2 : 当前已估计与未估计绝对旋转的顶点集合
 - $\mathcal{E}_{1,m}$: \mathcal{V}_2 中一顶点 v_m 与 \mathcal{V}_1 中所有顶点之间的边集
 - 对 $\mathcal{E}_{1,m}$ 中的每一条边 $e_{i,m}$ 预计算顶点 v_m 的绝对旋转 R_m^i
 - 对 v_m 的每一个预计算绝对旋转 R_m^i 获取支撑边集 $\mathcal{E}_{1,m}^i$



旋转平均 (Rotation Averaging, RA)

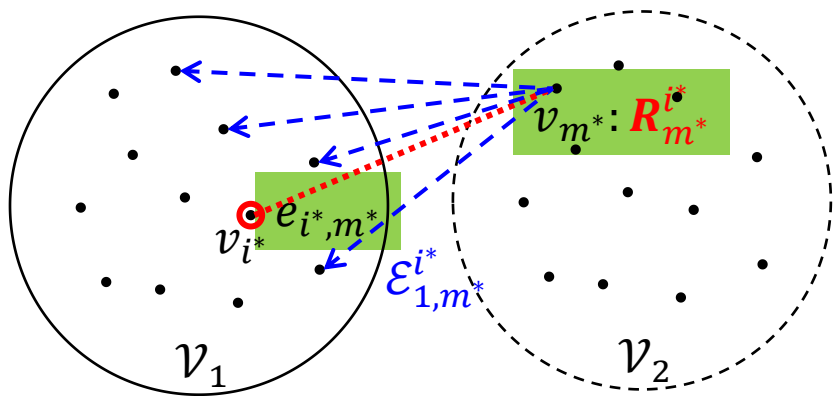
- IRA下一最优视图 (Next Best View, NBV) 选取与初始化
 - \mathcal{V}_1 与 \mathcal{V}_2 : 当前已估计与未估计绝对旋转的顶点集合
 - $\mathcal{E}_{1,m}$: \mathcal{V}_2 中一顶点 v_m 与 \mathcal{V}_1 中所有顶点之间的边集
 - 对 $\mathcal{E}_{1,m}$ 中的每一条边 $e_{i,m}$ 预计算顶点 v_m 的绝对旋转 R_m^i
 - 对 v_m 的每一个预计算绝对旋转 R_m^i 获取支撑边集 $\mathcal{E}_{1,m}^i$
 - 最大支撑边集 $\mathcal{E}_{1,m}^{i^*}$ 对应的 $R_m^{i^*}$ 即为 v_m 的初始化结果



旋转平均 (Rotation Averaging, RA)

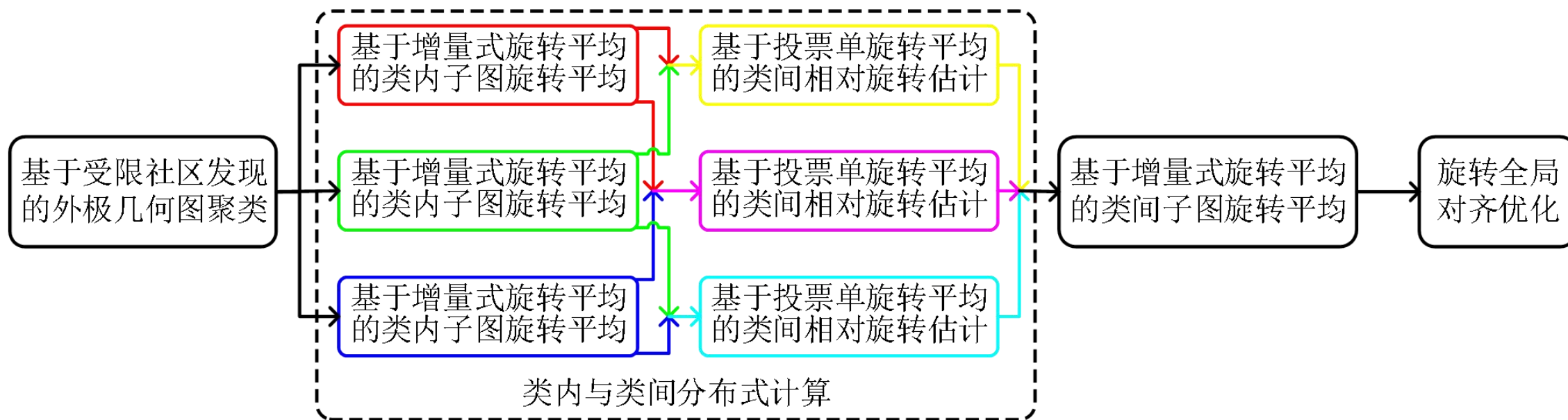
- IRA下一最优视图 (Next Best View, NBV) 选取与初始化

- \mathcal{V}_1 与 \mathcal{V}_2 : 当前已估计与未估计绝对旋转的顶点集合
- $\mathcal{E}_{1,m}$: \mathcal{V}_2 中一顶点 v_m 与 \mathcal{V}_1 中所有顶点之间的边集
- 对 $\mathcal{E}_{1,m}$ 中的每一条边 $e_{i,m}$ 预计算顶点 v_m 的绝对旋转 R_m^i
- 对 v_m 的每一个预计算绝对旋转 R_m^i 获取支撑边集 $\mathcal{E}_{1,m}^i$
- 最大支撑边集 $\mathcal{E}_{1,m}^{i^*}$ 对应的 $R_m^{i^*}$ 即为 v_m 的初始化结果
- 对 \mathcal{V}_2 中各顶点重复上述过程并获取全局最大支撑边集 $\mathcal{E}_{1,m^*}^{i^*}$, 则 v_{m^*} 与 $R_{m^*}^{i^*}$ 即为**NBV**及其**初始化**结果



旋转平均 (Rotation Averaging, RA)

- **层级增量式旋转平均 (Hierarchical Incremental Rotation Averaging, IRA++)**
 - 解决IRA因其固有的增量式参数估计流程在面向大规模旋转平均时的**累积误差与计算效率**问题

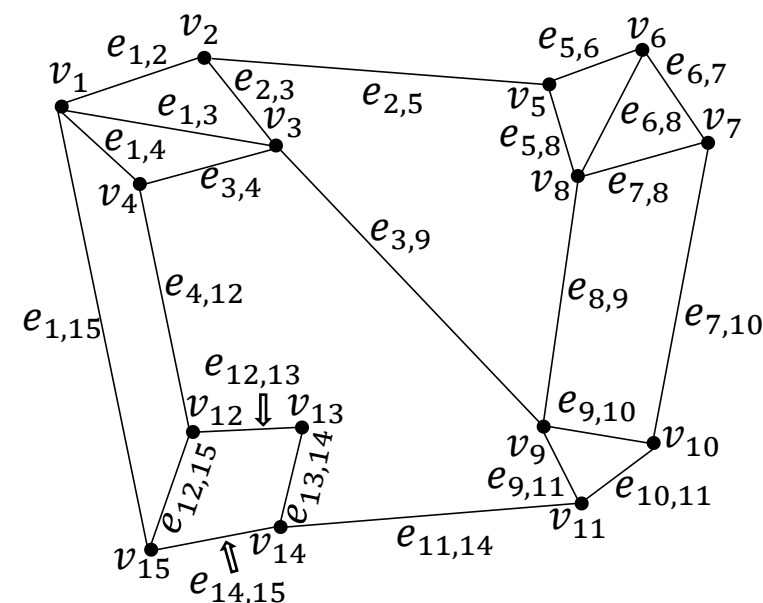


旋转平均 (Rotation Averaging, RA)

- 层级增量式旋转平均 (Hierarchical Incremental Rotation Averaging, IRA++)

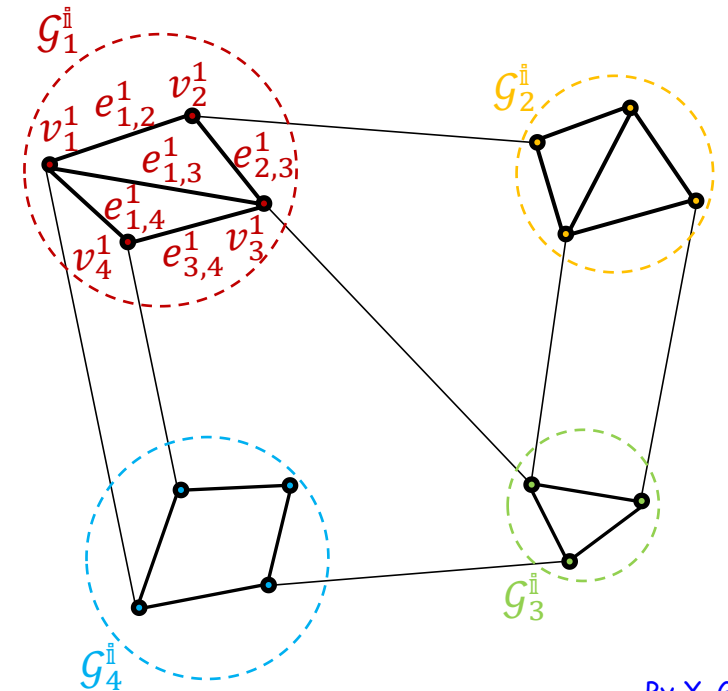
- 原始外极几何图:

- $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}, \mathcal{V} = \{v_i\}, \mathcal{E} = \{e_{i,j}\}$



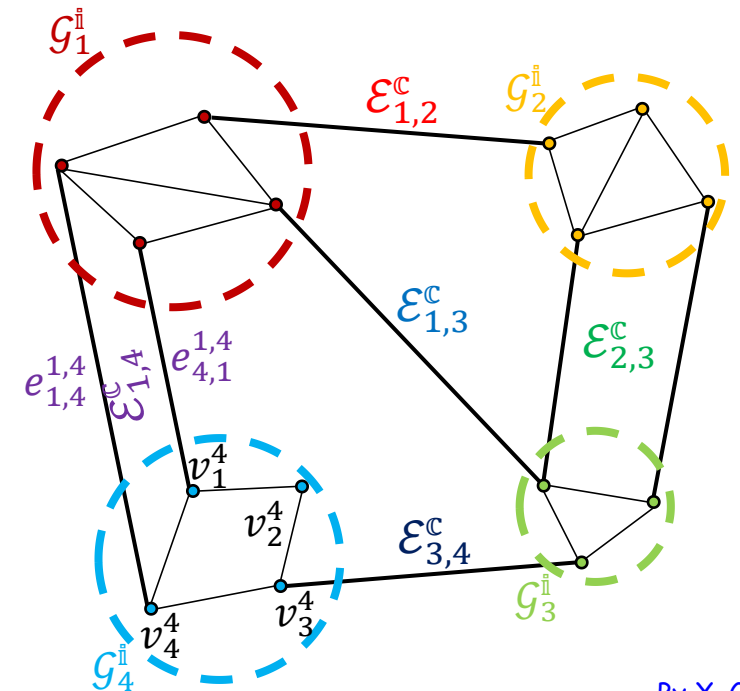
旋转平均 (Rotation Averaging, RA)

- 层级增量式旋转平均 (Hierarchical Incremental Rotation Averaging, IRA++)
 - 原始外极几何图
 - 基于社区发现的外极几何图聚类:
 - 类内子图: $\{\mathcal{G}_p^i = \{\mathcal{V}_p^i, \mathcal{E}_p^i\}\}$, $\mathcal{V}_p^i = \{v_m^p\}$, $\mathcal{E}_p^i = \{e_{m,n}^p\}$



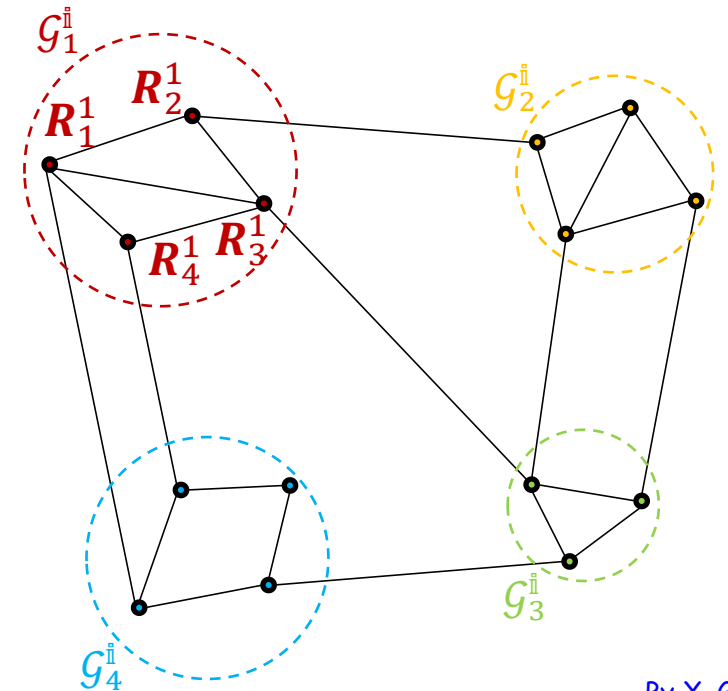
旋转平均 (Rotation Averaging, RA)

- 层级增量式旋转平均 (Hierarchical Incremental Rotation Averaging, IRA++)
 - 原始外极几何图
 - 基于社区发现的外极几何图聚类:
 - 类内子图: $\{G_p^i = \{V_p^i, \mathcal{E}_p^i\}\}$, $V_p^i = \{v_m^p\}$, $\mathcal{E}_p^i = \{e_{m,n}^p\}$
 - 类间子图: $G^c = \{V^c, \mathcal{E}^c\}$, $V^c = \{G_p^i\}$, $\mathcal{E}^c = \{\mathcal{E}_{p,q}^c\} = \{\{e_{m,n}^{p,q}\}\}$



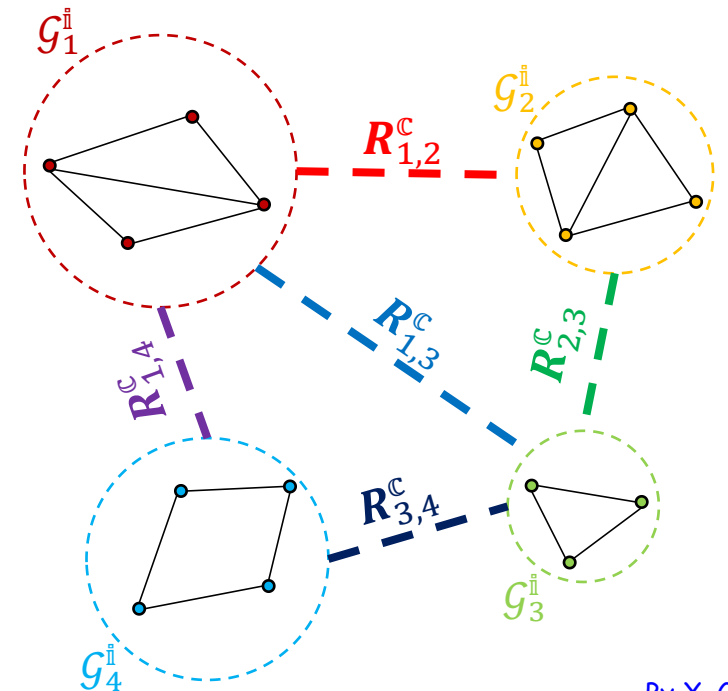
旋转平均 (Rotation Averaging, RA)

- 层级增量式旋转平均 (Hierarchical Incremental Rotation Averaging, IRA++)
 - 原始外极几何图
 - 基于社区发现的外极几何图聚类
 - 基于IRA的类内旋转平均:
 - $\{\mathbf{R}_{m,n}^p \mid e_{m,n}^p \in \mathcal{E}_p^i\} \Rightarrow \{\mathbf{R}_m^p \mid v_m^p \in \mathcal{V}_p^i\}$



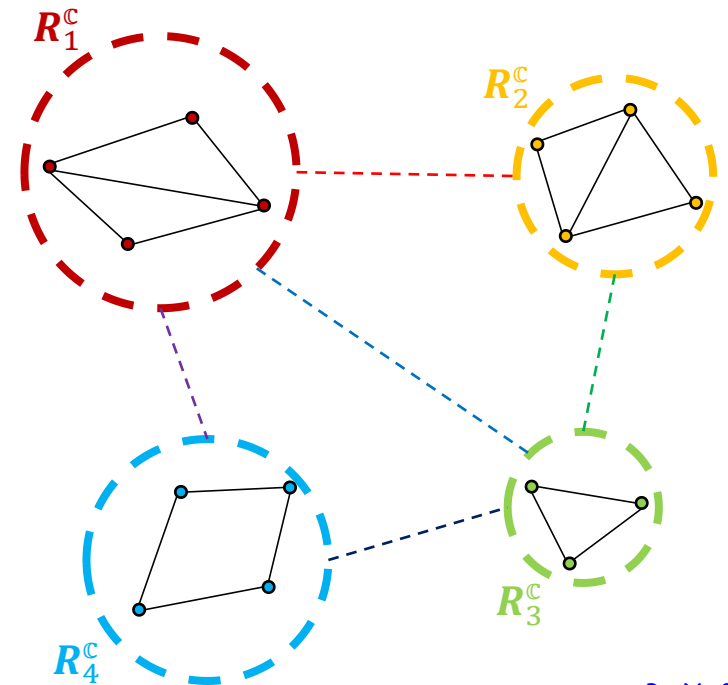
旋转平均 (Rotation Averaging, RA)

- 层级增量式旋转平均 (Hierarchical Incremental Rotation Averaging, IRA++)
 - 原始外极几何图
 - 基于社区发现的外极几何图聚类
 - 基于IRA的类内旋转平均
 - 基于单旋转平均的类间相对旋转估计:
 - $\{\mathbf{R}_{m,n}^{p,q} | e_{m,n}^{p,q} \in \mathcal{E}_{p,q}\} \Rightarrow \{\mathbf{R}_{p,q}^c | \mathcal{E}_{p,q}^c \in \mathcal{E}^c\}$



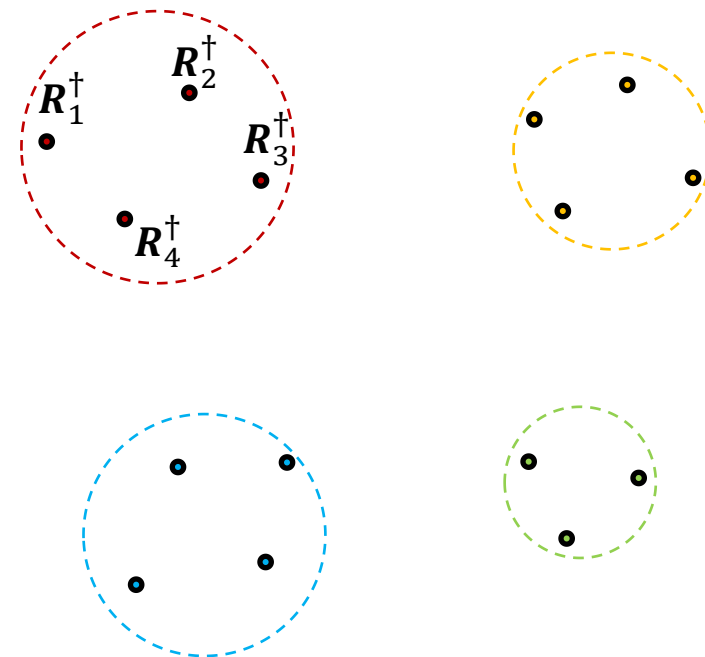
旋转平均 (Rotation Averaging, RA)

- 层级增量式旋转平均 (Hierarchical Incremental Rotation Averaging, IRA++)
 - 原始外极几何图
 - 基于社区发现的外极几何图聚类
 - 基于IRA的类内旋转平均
 - 基于单旋转平均的类间相对旋转估计
 - 基于IRA的类间旋转平均:
 - $\{R_{p,q}^c | \mathcal{E}_{p,q}^c \in \mathcal{E}^c\} \Rightarrow \{R_p^c | \mathcal{G}_p^i \in \mathcal{V}^c\}$



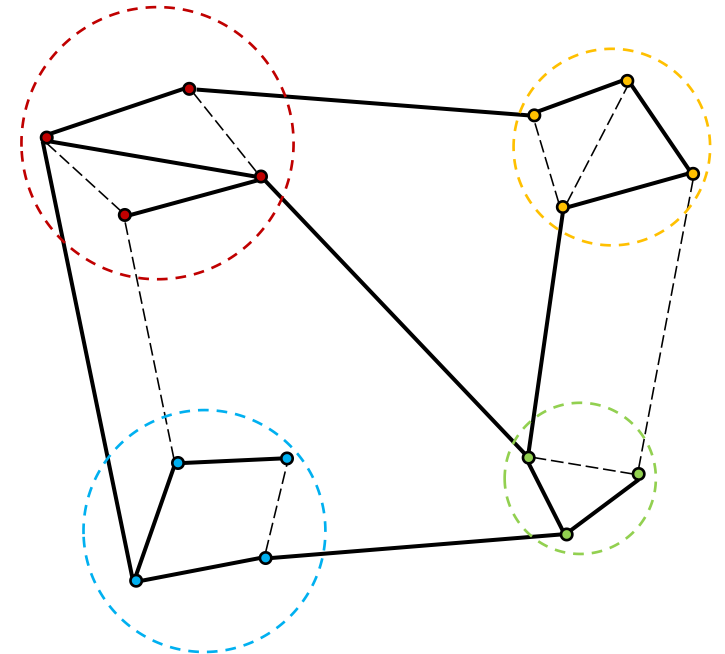
旋转平均 (Rotation Averaging, RA)

- 层级增量式旋转平均 (Hierarchical Incremental Rotation Averaging, IRA++)
 - 原始外极几何图
 - 基于社区发现的外极几何图聚类
 - 基于IRA的类内旋转平均
 - 基于单旋转平均的类间相对旋转估计
 - 基于IRA的类间旋转平均
 - 绝对旋转全局对齐与优化:
 - 全局对齐: $R_i^\dagger = R_m^p R_p^c$



旋转平均 (Rotation Averaging, RA)

- 层级增量式旋转平均 (Hierarchical Incremental Rotation Averaging, IRA++)
 - 原始外极几何图
 - 基于社区发现的外极几何图聚类
 - 基于IRA的类内旋转平均
 - 基于单旋转平均的类间相对旋转估计
 - 基于IRA的类间旋转平均
 - 绝对旋转全局对齐与优化:
 - 全局对齐: $\mathbf{R}_i^\dagger = \mathbf{R}_m^p \mathbf{R}_p^c$
 - 内值计算: $\mathcal{E}^\mathbb{I} = \{d_\theta^{\mathbf{R}}(\mathbf{R}_{i,j}, \mathbf{R}_j^\dagger \mathbf{R}_i^{\dagger\top}) < \theta_{th}^{\mathbf{R}}\}$ for $v_i, v_j \in \mathcal{V}, e_{i,j} \in \mathcal{E}$



旋转平均 (Rotation Averaging, RA)

- 层级增量式旋转平均 (Hierarchical Incremental Rotation Averaging, IRA++)

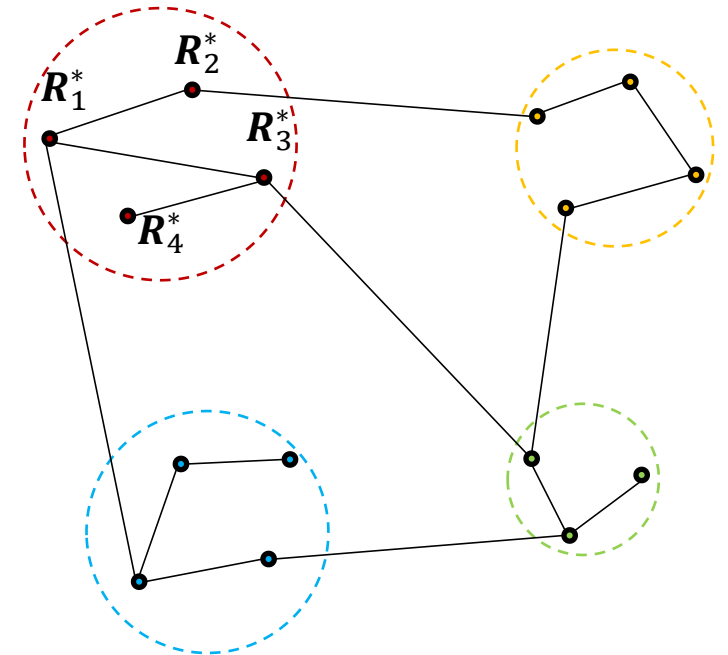
- 原始外极几何图
- 基于社区发现的外极几何图聚类
- 基于IRA的类内旋转平均
- 基于单旋转平均的类间相对旋转估计
- 基于IRA的类间旋转平均

- 绝对旋转全局对齐与优化:

- 全局对齐: $R_i^\dagger = R_m^p R_p^c$

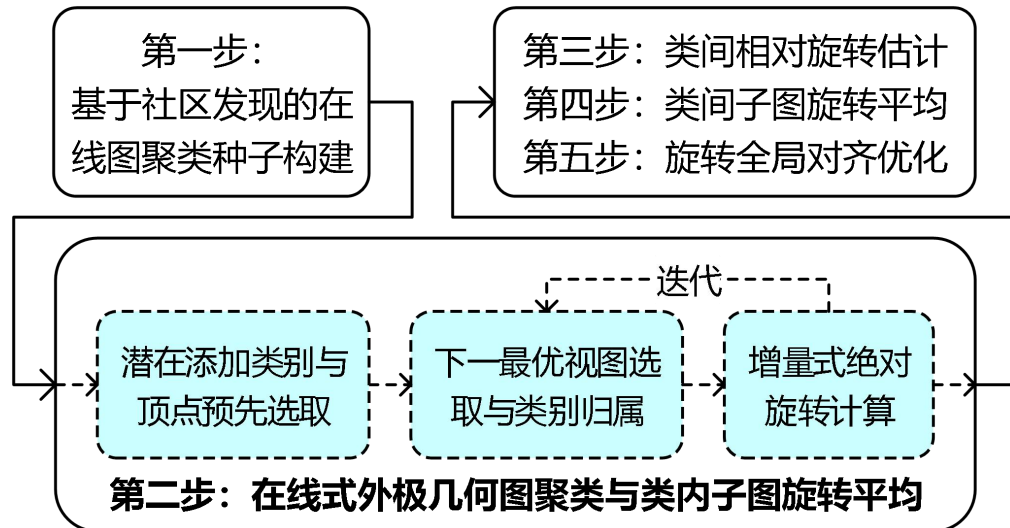
- 内值计算: $\mathcal{E}^\mathbb{I} = \{d_\theta^R(R_{i,j}, R_j^\dagger R_i^{\dagger T}) < \theta_{th}^R\}$ for $v_i, v_j \in \mathcal{V}, e_{i,j} \in \mathcal{E}$

- 全局优化: $\{R_i^*\} = \arg \min_{e_{i,j} \in \mathcal{E}^\mathbb{I}} \sum_{v_i, v_j \in \mathcal{V}} d_\theta^R(R_{i,j}^\mathbb{I}, R_j R_i^\top)^2$



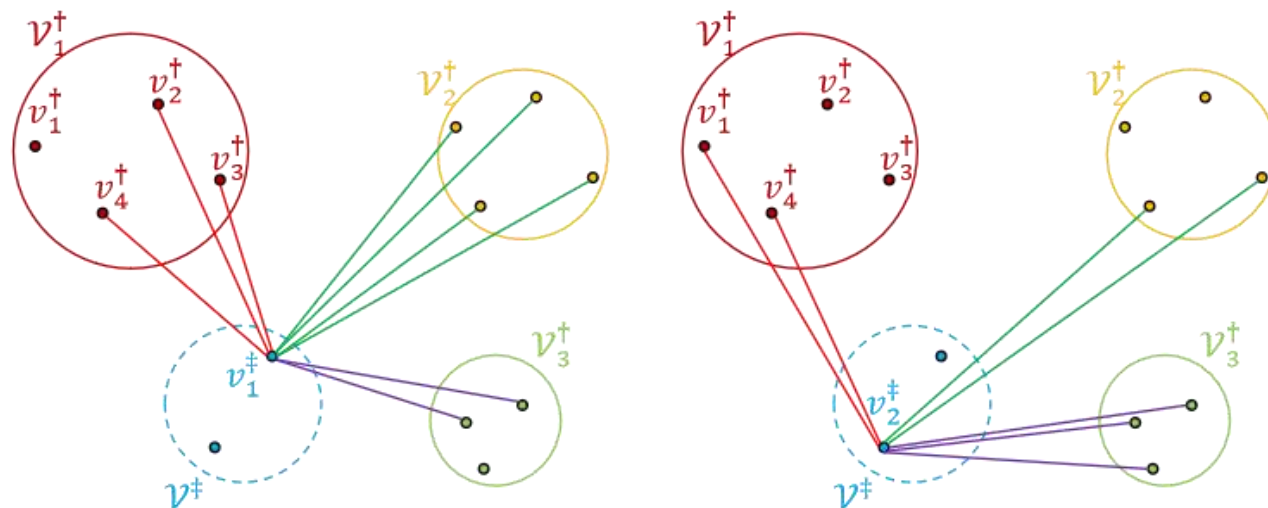
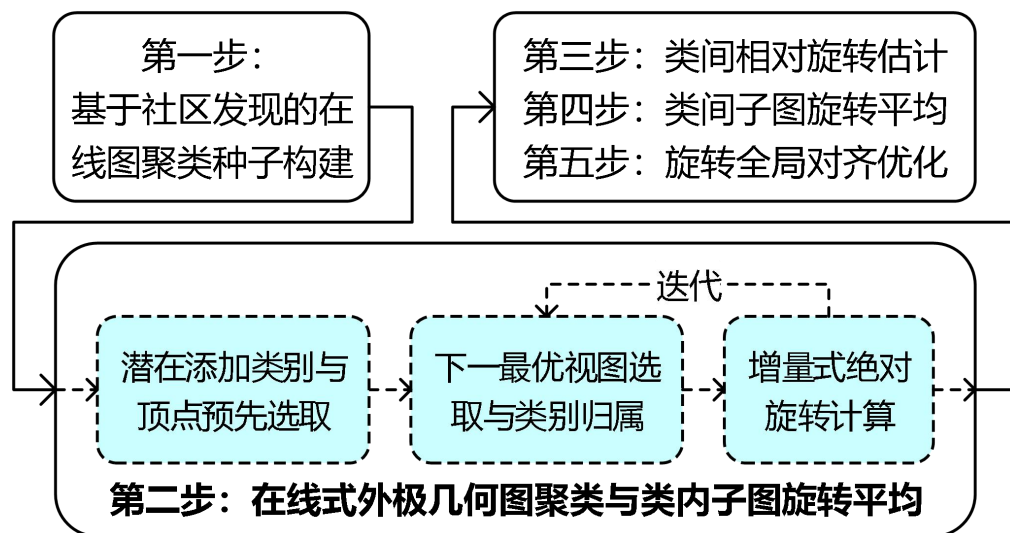
旋转平均 (Rotation Averaging, RA)

- **在线**层级增量式旋转平均
- Hierarchical Incremental Rotation Averaging **on the Fly**, IRAv3
 - 解决**IRA++**中基于社区发现的聚类方式所存在的**预先确定与任务无关**问题



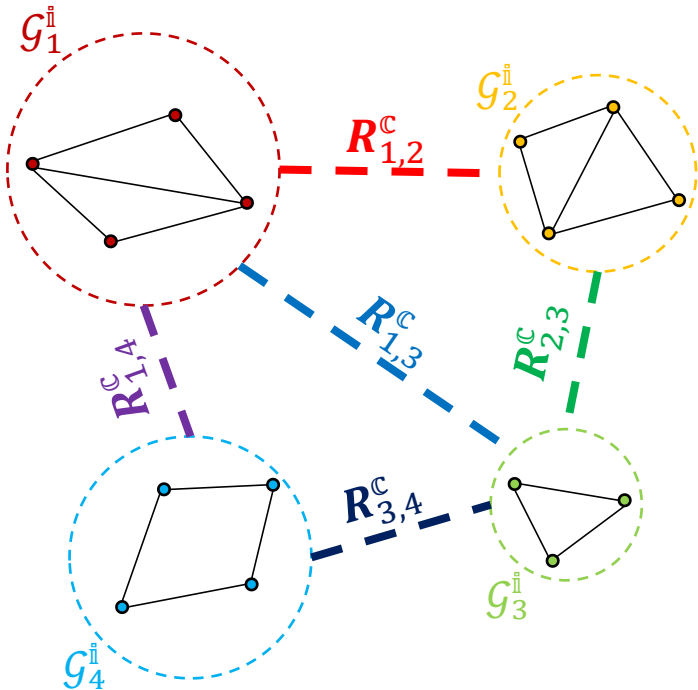
旋转平均 (Rotation Averaging, RA)

- **在线**层级增量式旋转平均
- Hierarchical Incremental Rotation Averaging **on the Fly**, IRAv3
 - 解决IRA++中基于社区发现的聚类方式所存在的**预先确定与任务无关**问题
 - \mathcal{V}_i^+ : 当前已估计绝对旋转的各类别顶点集合
 - \mathcal{V}^\ddagger : 当前未估计绝对旋转的顶点集合
 - 图中实线: \mathcal{V}^\ddagger 中顶点 v_j^\ddagger 在 \mathcal{V}_i^+ 中的支撑边集



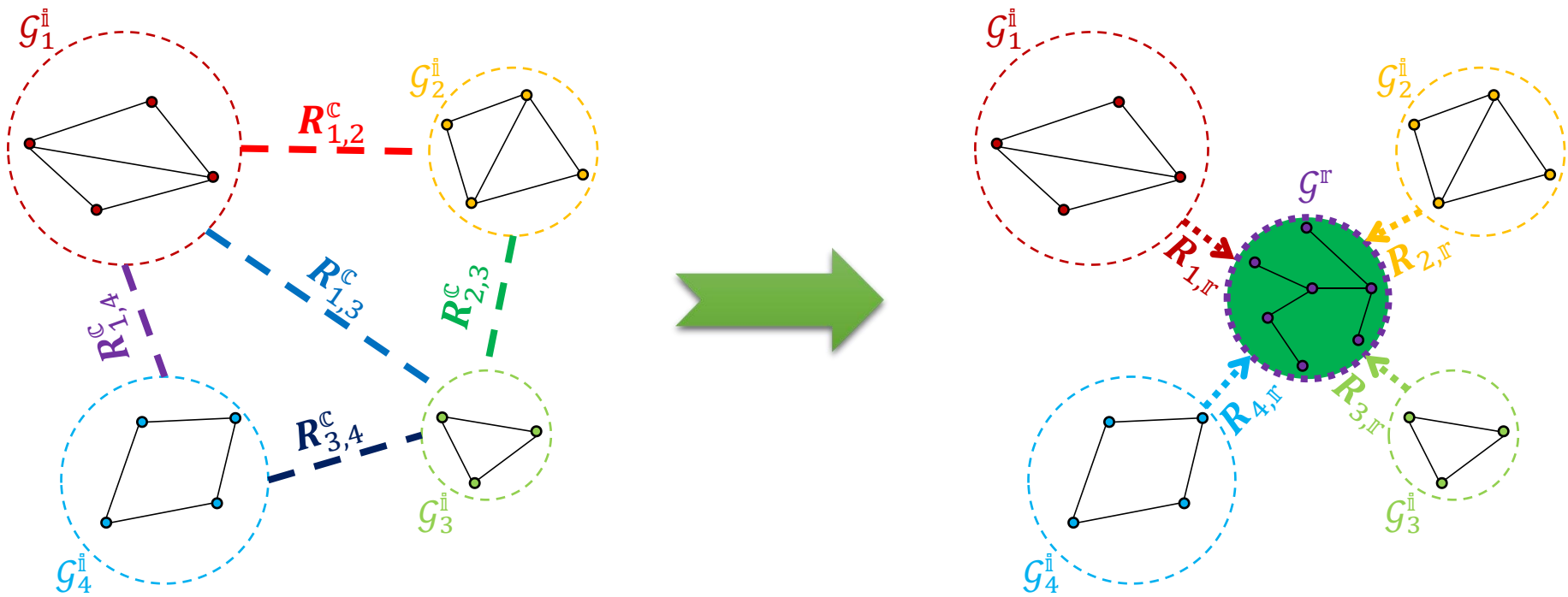
旋转平均 (Rotation Averaging, RA)

- 基准层级增量式旋转平均
- Hierarchical Incremental Rotation Averaging via **Multiple Connected Dominating Sets**, IRAv3+
 - 解决IRA++/IRA_{v3}中基于类间旋转平均的全局对齐存在的**类间连接弱**与**对齐精度低**的问题



旋转平均 (Rotation Averaging, RA)

- 基准层级增量式旋转平均
- Hierarchical Incremental Rotation Averaging via **Multiple Connected Dominating Sets, IRAv3+**
 - 解决IRA++/IRA_{v3}中基于类间旋转平均的全局对齐存在的**类间连接弱与对齐精度低**的问题
 - 连通支配集：原始图的连通子集，且原始图中的全部顶点或属于该子集或与该子集某顶点相邻



旋转平均 (Rotation Averaging, RA)

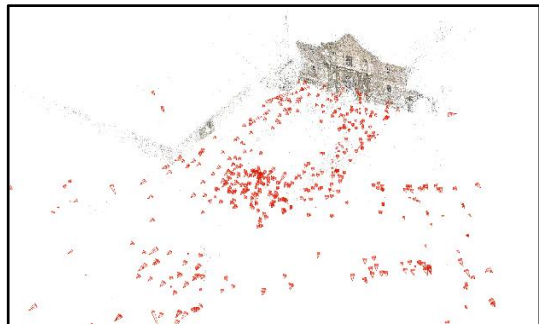
- 实验数据: 1DSfM

	$ \mathcal{V} $	$ \mathcal{V}_{GT} $	$ \mathcal{E} $	$\tilde{n}_{i,j}$	$\bar{n}_{i,j}$	$\tilde{r}_{i,j}^R$	$\bar{r}_{i,j}^R$	$\tilde{r}_{i,j}^t$	$\bar{r}_{i,j}^t$
ALM	627	577	97206	105	192	2.78°	9.09°	4.65°	18.80°
ELS	247	227	20297	106	160	2.89°	12.50°	8.75°	36.66°
GDM	742	677	48144	73	144	12.30°	33.33°	26.87°	52.65°
MDR	394	341	23784	61	128	9.34°	29.30°	13.46°	36.30°
MND	474	450	52424	180	310	1.67°	7.51°	3.33°	18.61°
NYC	376	332	20680	80	167	4.22°	14.14°	7.18°	28.86°
PDP	354	338	24710	87	128	1.81°	8.38°	3.07°	21.75°
PIC	2508	2152	319257	56	97	4.93°	19.09°	2.92°	7.71°
ROF	1134	1084	70187	65	188	2.97°	13.83°	4.01°	30.19°
TOL	508	472	23863	81	220	2.60°	11.58°	2.63°	19.92°
TFG	5433	5058	680012	71	109	3.01°	8.62°	6.56°	23.67°
USQ	930	789	25561	87	150	3.61°	9.02°	20.32°	43.22°
VNC	918	836	103550	229	408	2.59°	11.26°	4.24°	24.47°
YKM	458	437	27729	112	245	2.68°	11.16°	3.40°	21.20°

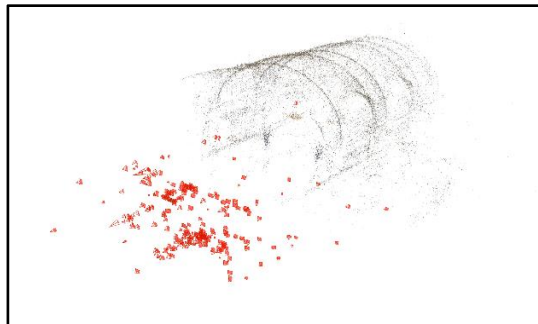
- $|\mathcal{V}|$, $|\mathcal{E}|$, $|\mathcal{V}_{GT}|$: 外极几何图顶点数与边数以及带有真值的顶点数
- $\tilde{n}_{i,j}$, $\bar{n}_{i,j}$: 匹配图像对之间的图像局部特征匹配对数的中值与均值
- $\tilde{r}_{i,j}^R$, $\bar{r}_{i,j}^R$: 匹配图像对之间的相对旋转测量值的角度误差中值与均值
- $\tilde{r}_{i,j}^t$, $\bar{r}_{i,j}^t$: 匹配图像对之间的相对平移测量值的角度误差中值与均值

旋转平均 (Rotation Averaging, RA)

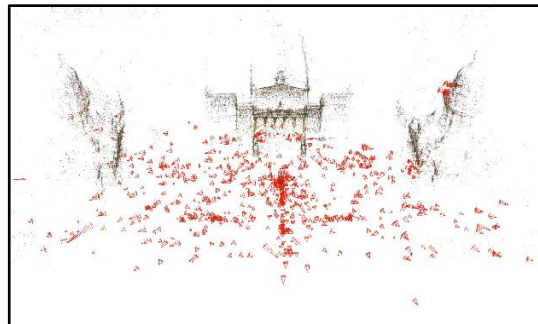
- 实验数据: 1DSfM



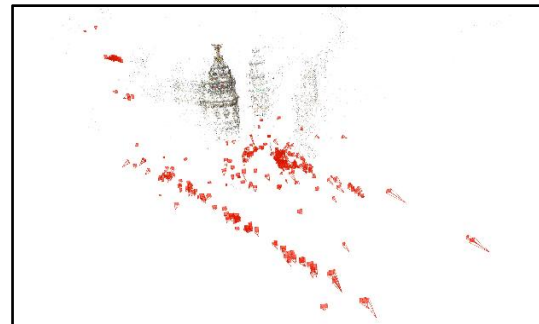
ALM



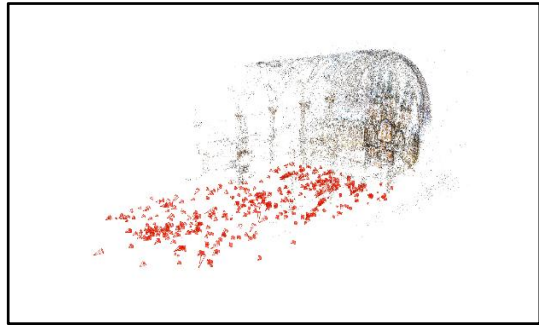
ELS



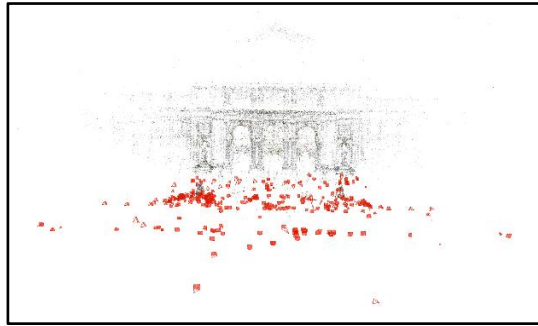
GDM



MDR



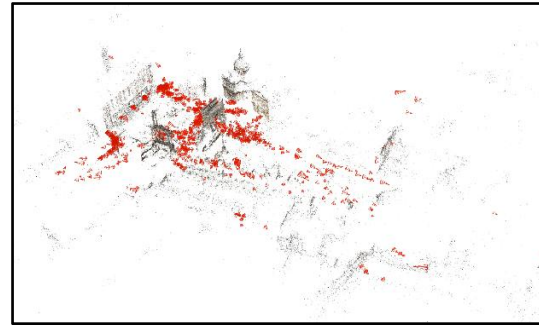
MND



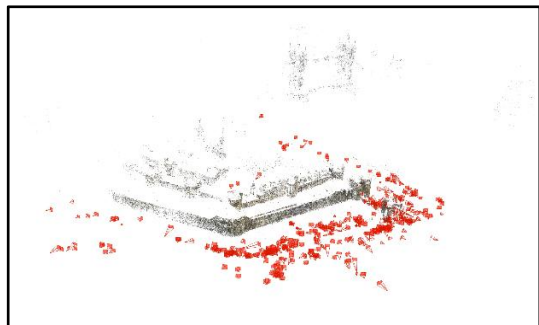
NYC



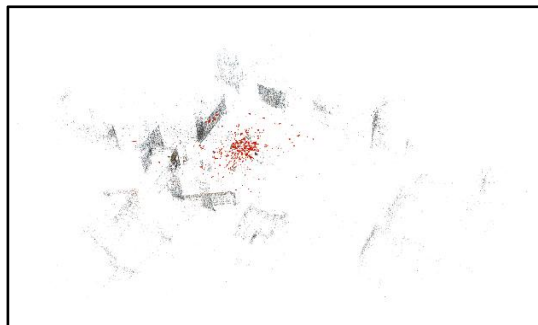
PDP



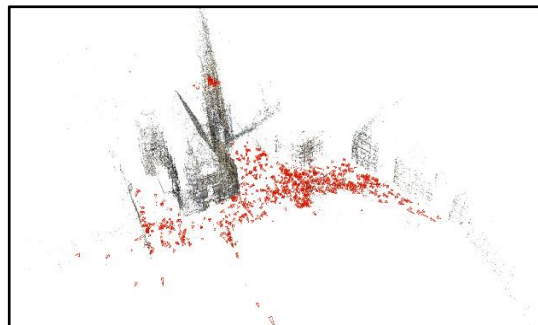
ROF



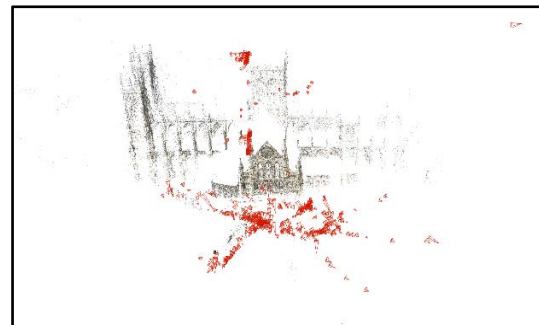
TOL



USQ



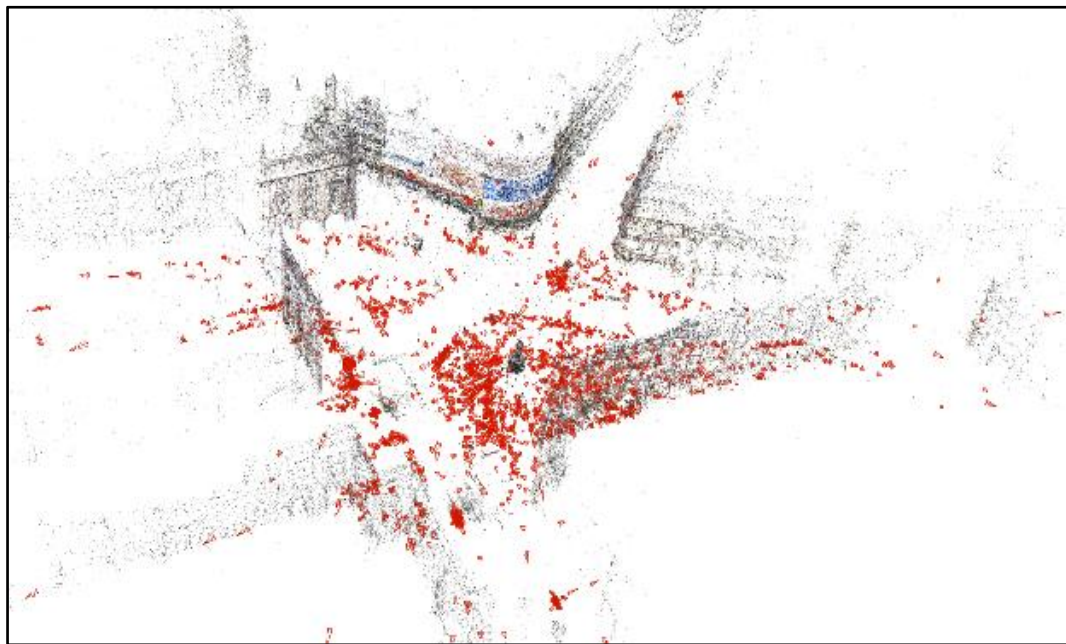
PDP



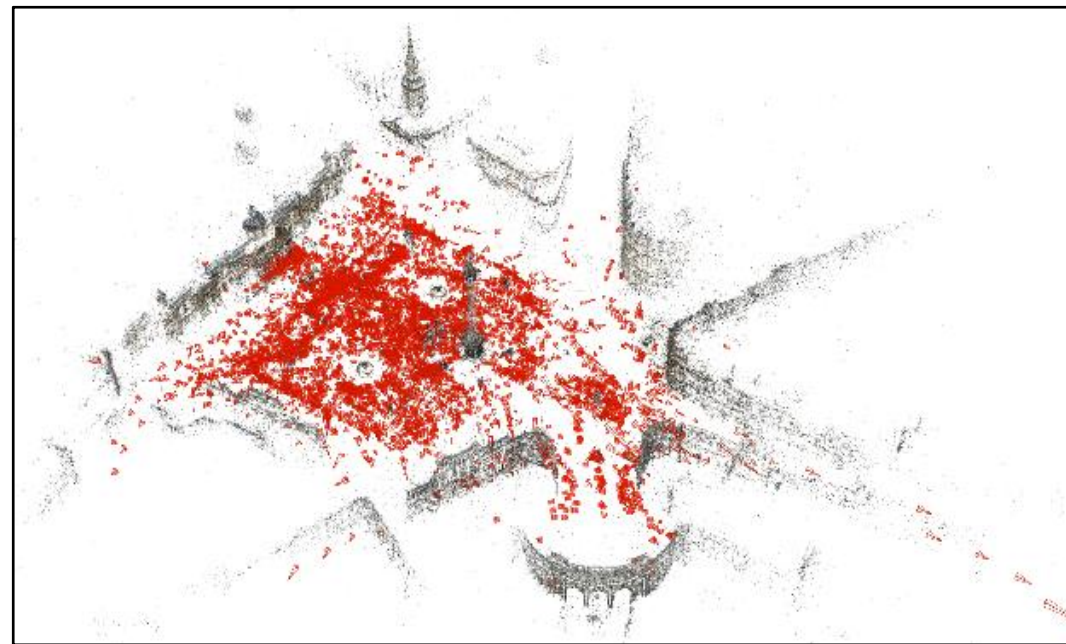
ROF

旋转平均 (Rotation Averaging, RA)

- 实验数据: 1DSfM



PIC



TFG

旋转平均 (Rotation Averaging, RA)

• 实验结果

Data	Robust loss-based			Outlier filtering-based			Deep learning-based			IRA series			
	IRLS- $\ell_{\frac{1}{2}}$ [1]	MPLS [2]	DESC [3]	OMSTs [4]	HARRA [5]	HARA [6]	NeuRoRA [7]	MSP [8]	RAGO [9]	IRA	IRA++	IRAv3	IRA3v+
ALM	2.14°	1.16°	1.63°	1.26°	1.03°	1.15°	1.16°	1.07°	0.88°	0.83°	0.80°	0.73°	0.72°
ELS	1.15°	0.88°	1.01°	0.75°	0.59°	0.62°	0.64°	0.83°	0.46°	0.51°	0.46°	0.44°	0.39°
GDM	28.20°	9.87°	9.43°	45.15°	4.04°	63.74°	2.94°	3.69°	2.68°	5.32°	2.88°	1.99°	2.20°
MDR	3.08°	1.26°	2.35°	1.12°	2.54°	1.50°	1.13°	1.09°	1.03°	0.85°	0.83°	0.75°	0.75°
MND	0.71°	0.51°	0.59°	0.68°	0.62°	0.51°	0.60°	0.50°	0.46°	0.51°	0.50°	0.44°	0.40°
NYC	1.40°	1.24°	1.48°	1.30°	1.24°	1.37°	1.18°	1.12°	0.71°	1.00°	0.95°	0.82°	0.81°
PDP	2.62°	1.93°	1.95°	1.73°	0.92°	0.92°	0.79°	0.76°	0.63°	0.90°	0.75°	0.72°	0.63°
PIC	3.12°	1.81°	2.43°	1.41°	4.87°	3.22°	1.91°	1.80°	0.58°	1.67°	1.70°	1.50°	1.55°
ROF	1.70°	1.37°	1.42°	1.85°	2.48°	2.42°	1.31°	1.19°	1.10°	1.51°	1.24°	1.09°	1.14°
TOL	2.45°	2.20°	2.49°	2.10°	2.05°	2.36°	1.46°	1.25°	1.20°	2.45°	1.33°	1.44°	1.09°
TFG	2.03°	—	1.84°	2.63°	4.88°	2.06°	2.25°	—	1.53°	3.30°	1.74°	1.49°	1.80°
USQ	4.97°	3.48°	4.31°	3.83°	3.77°	4.78°	2.01°	1.85°	1.92°	4.40°	3.70°	3.27°	2.77°
VNC	4.64°	2.83°	1.99°	3.30°	1.84°	1.49°	1.50°	1.10°	0.89°	1.02°	0.94°	0.86°	0.76°
YKM	1.62°	1.45°	1.60°	1.55°	1.57°	1.65°	0.99°	0.91°	0.92°	1.57°	1.38°	1.36°	0.98°
Rank	11.71	8.62	10.36	9.29	9.14	9.71	6.93	5.08	2.50	7.07	4.29	2.57	2.07

[1] A. Chatterjee and V. M. Govindu. [Robust Relative Rotation Averaging](#)[J]. *IEEE T-PAMI*, 2018.

[2] Y. Shi and G. Lerman. [Message Passing Least Squares Framework and its Application to Rotation Synchronization](#)[C]. *In Proc. ICML*, 2020.

[3] Y. Shi, C. M. Wyeth, and G. Lerman. [Robust Group Synchronization via Quadratic Programming](#)[C]. *In Proc. ICML*, 2022.

[4] H. Cui, S. Shen, W. Gao, L. Hong, and Z. Wang. [Efficient and Robust Large-Scale Structure-from-Motion via Track Selection and Camera Prioritization](#)[J]. *ISPRS P&RS*, 2019.

[5] X. Gao, J. Luo, K. Li, and Z. Xie. [Hierarchical RANSAC-Based Rotation Averaging](#)[J]. *IEEE SPL*, 2020.

[6] S. H. Lee and J. Civera. [HARA: A Hierarchical Approach for Robust Rotation Averaging](#)[C]. *In Proc. CVPR*, 2022.

[7] P. Purkait, T. J. Chin, and I. Reid. [NeuRoRA: Neural Robust Rotation Averaging](#)[C]. *In Proc. ECCV*, 2020.

[8] L. Yang, H. Li, J. A. Rahim, Z. Cui, and P. Tan. [End-to-End Rotation Averaging with Multi-Source Propagation](#)[C]. *In Proc. CVPR*, 2021.

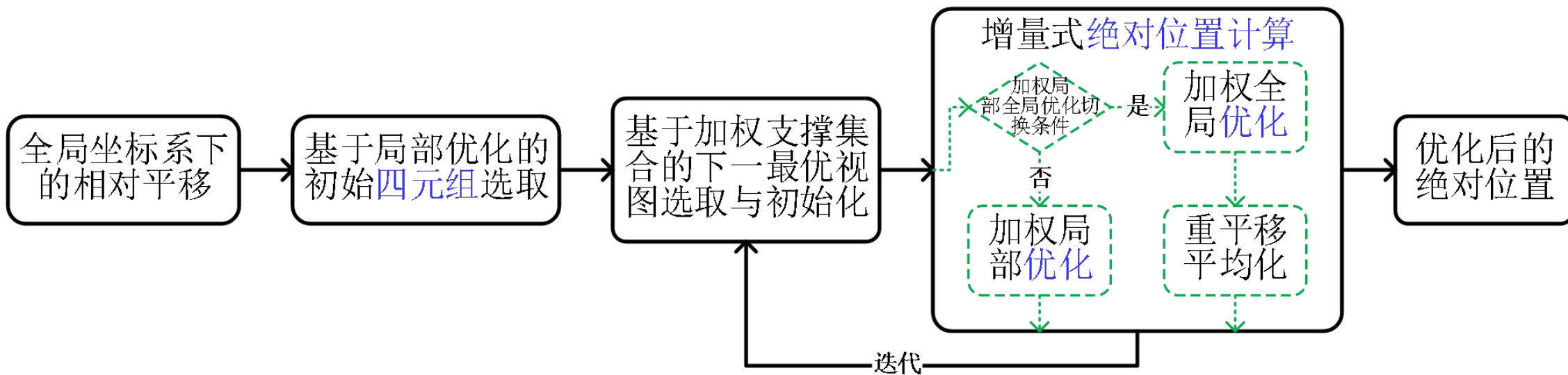
[9] H. Li, Z. Cui, S. Liu, and P. Tan. [RAGO: Recurrent Graph Optimizer for Multiple Rotation Averaging](#)[C]. *In Proc. CVPR*, 2022.

平移平均 (Translation Averaging, TA)

- 增量式平移平均 (Incremental Translation Averaging, ITA)

- IRA \Rightarrow ITA

- 相机位置算子
- 流程关键技术
- 优化目标函数



平移平均 (Translation Averaging, TA)

- 增量式平移平均 (Incremental Translation Averaging, ITA)

- IRA \Rightarrow ITA

- 相机位置算子

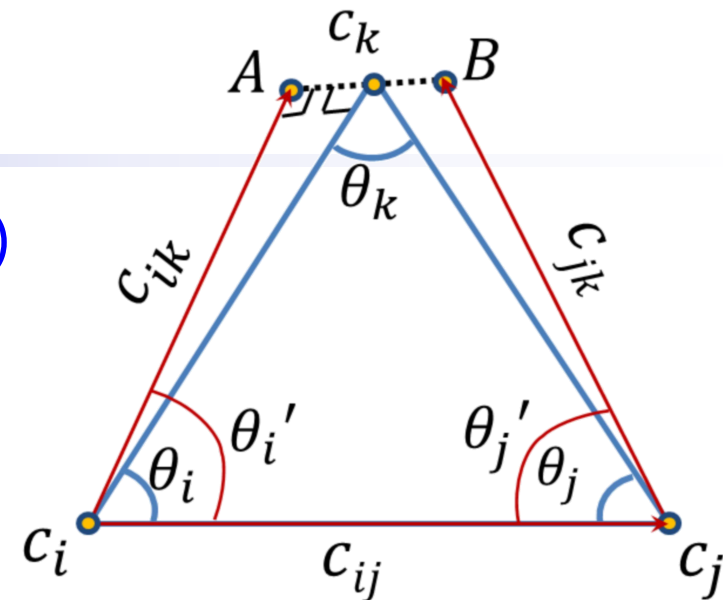
IRA, 已知 R_i 、 $R_{i,j}$, 求 R_j : $R_j = R_{i,j}R_i$

ITA, 已知 c_i 、 c_j 、 $c_{i,j}$ 、 $c_{i,k}$ 、 $c_{j,k}$, 求 c_k :

$$c_k = \frac{1}{2} \left(R_i(\theta'_i) s_{i,j}^{i,k} (c_j - c_i) + R_j(-\theta'_j) s_{i,j}^{j,k} (c_i - c_j) + c_i + c_j \right)$$

- 流程关键技术

- 优化目标函数



Jiang et al. In Proc. ICCV 2013

平移平均 (Translation Averaging, TA)

- 增量式平移平均 (Incremental Translation Averaging, ITA)

- IRA \Rightarrow ITA

- 相机位置算子
- 流程关键技术

IRA, 基于局部优化的初始三元组选取:

$$i^*, j^*, k^* = \arg \max \left\{ \sum_{\substack{v_i, v_j \in \mathcal{V}_{t_{i,j,k}} \\ e_{i,j} \in \mathcal{E}_{t_{i,j,k}}} n_{i,j} \cos \left(d_{\theta}^R(\mathbf{R}_{i,j}, \mathbf{R}_j^* \mathbf{R}_i^{*\top}) \right) \mid t_{i,j,k} \in \mathcal{T}^* \right\}$$

ITA, 基于局部优化的初始四元组选取:

$$i^*, j^*, k^*, l^* = \arg \max \left\{ \sum_{\substack{v_i, v_j \in \mathcal{V}_{q_{i,j,k,l}} \\ e_{i,j} \in \mathcal{E}_{q_{i,j,k,l}}} \mathbf{c}_{i,j} \cdot \frac{\mathbf{c}_j^* - \mathbf{c}_i^*}{\|\mathbf{c}_j^* - \mathbf{c}_i^*\|_2} \mid q_{i,j,k,l} \in \mathcal{Q}^* \right\}$$

- 优化目标函数

平移平均 (Translation Averaging, TA)

- 增量式平移平均 (Incremental Translation Averaging, ITA)
 - IRA \Rightarrow ITA
 - 相机位置算子
 - 流程关键技术
 - 优化目标函数

IRA, 旋转矩阵角距离最小化: $\{\mathbf{R}_i^*\} = \arg \min_{e_{i,j} \in \mathcal{E}} \sum_{v_i, v_j \in \mathcal{V}} d_{\theta}^{\mathbf{R}}(\mathbf{R}_{i,j}, \mathbf{R}_j \mathbf{R}_i^{\top})^2$

ITA, 位置向量弦距离最小化: $\{\mathbf{c}_i^*\} = \arg \min_{e_{i,j} \in \mathcal{E}} \sum_{v_i, v_j \in \mathcal{V}} d_{ch}^t\left(\mathbf{c}_{i,j}, \frac{\mathbf{c}_j - \mathbf{c}_i}{\|\mathbf{c}_j - \mathbf{c}_i\|_2}\right)^2$

平移平均 (Translation Averaging, TA)

- 增量式平移平均 (Incremental Translation Averaging, ITA)

- 实验结果

Data	# reconstructed cameras estimation error/m elapsed time/s on different translation averaging methods							
	SATA[1]	VGO[2]	SFSK [3]	BATA[4]	ACEM[5]	Our ITA	ITA w/ IRA	ITA w/ IRA*
ALM	574 0.5 78	533 1.4 69	- 0.9 27	- 0.6 24	482 1.2 172	575 0.5 17	577 0.5 17	523 0.4 10
ELS	223 2.5 37	203 3.7 19	- 1.9 4	- 1.5 2	211 6.1 87	230 1.3 2	227 1.1 2	224 1.0 2
MDR	317 2.7 31	272 8.7 67	- 6.0 9	- 1.8 4	168 6.9 23	339 7.0 3	336 6.5 4	294 1.5 2
MND	452 0.4 62	416 2.0 133	- 0.8 19	- 0.3 10	416 1.0 224	446 0.4 10	452 0.4 10	429 0.4 9
NYC	338 0.8 38	294 2.8 71	- 1.4 11	- 0.6 4	277 2.2 67	327 0.6 3	328 0.7 4	304 0.4 3
PDP	340 2.0 43	302 2.9 23	- 3.6 7	- 4.2 4	275 3.5 60	326 0.6 5	322 0.6 4	299 0.5 5
PIC	2276 1.3 328	1928 5.2 544	- 1.2 464	- 1.0 114	- - -	2199 1.8 381	2231 1.5 402	1929 0.7 268
ROF	1077 2.9 131	966 6.8 385	- 4.3 62	- 1.6 29	- - -	1062 2.3 28	1053 2.6 27	997 2.2 22
TOL	465 1.9 52	409 9.3 145	- 2.3 18	- 2.2 7	414 5.0 121	453 1.8 7	452 2.0 7	422 1.9 5
USQ	570 5.5 41	701 4.5 141	- 8.9 28	- 4.3 10	- - -	703 5.6 8	697 5.3 9	588 3.0 6
VNC	842 2.7 117	771 6.7 185	- 1.9 74	- 1.9 30	674 4.2 273	783 1.0 27	778 1.2 26	720 0.8 17
YKM	417 2.3 46	409 3.9 31	- - -	- 0.9 9	341 2.7 91	414 0.8 5	411 0.8 6	385 0.7 6

[1] Z. Cui and P. Tan. [Global Structure-from-Motion by Similarity Averaging](#)[C]. *In Proc. ICCV*, 2015.

[2] C. Sweeney, T. Sattler, T. Höllerer, M. Turk, and M. Pollefeys. [Optimizing the Viewing Graph for Structure-from-Motion](#)[C]. *In Proc. ICCV*, 2015.

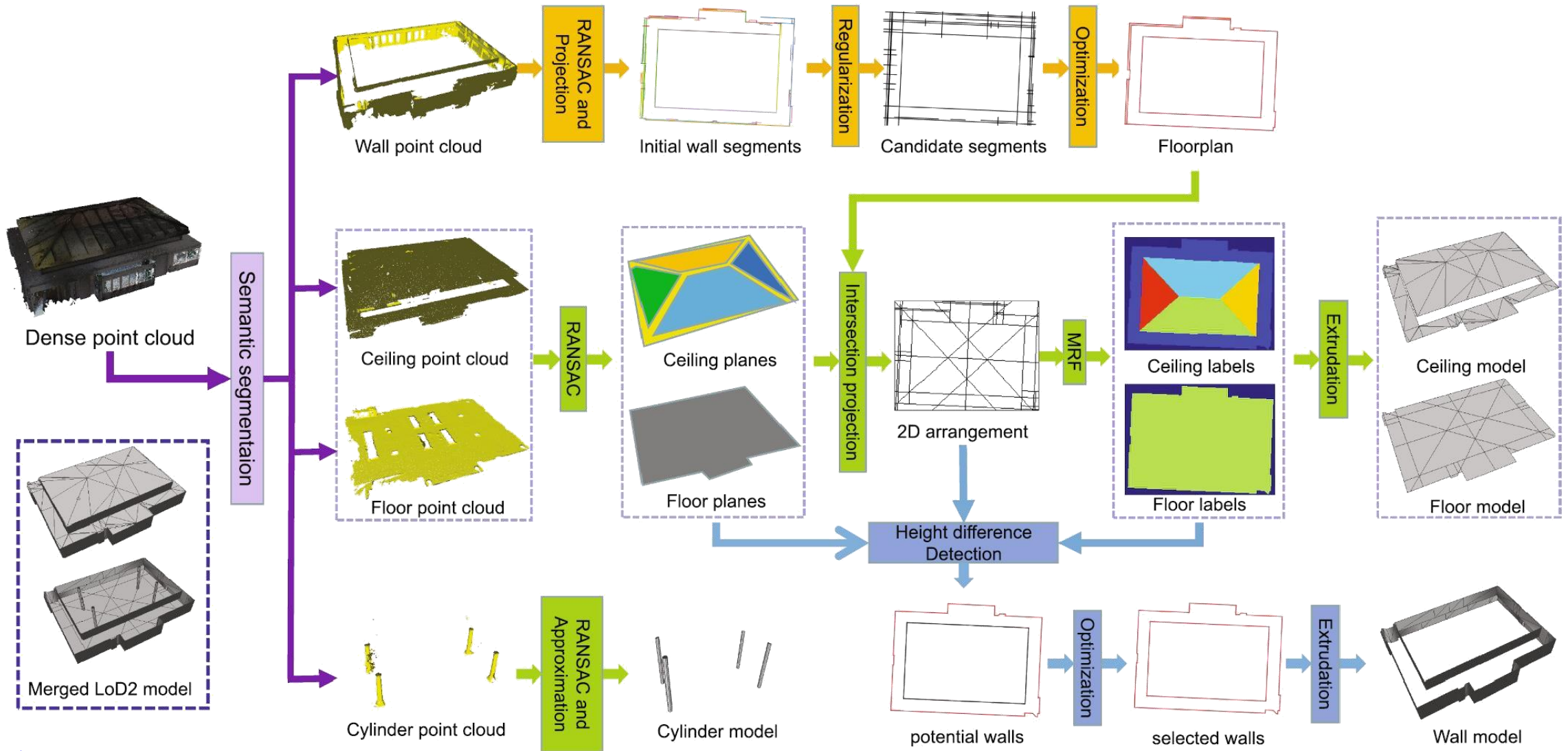
[3] T. Goldstein, P. Hand, C. Lee, V. Voroninski, and S. Soatto. [ShapeFit and ShapeKick for Robust, Scalable Structure from Motion](#)[C]. *In Proc. ECCV*, 2016.

[4] B. Zhuang, L. Cheong, and G. H. Lee. [Baseline Desensitizing in Translation Averaging](#)[C]. *In Proc. CVPR*, 2018.

[5] Y. Kasten, A. Geifman, M. Galun, and R. Basri. [Algebraic Characterization of Essential Matrices and Their Averaging in Multiview Settings](#)[C]. *In Proc. ICCV*, 2019.

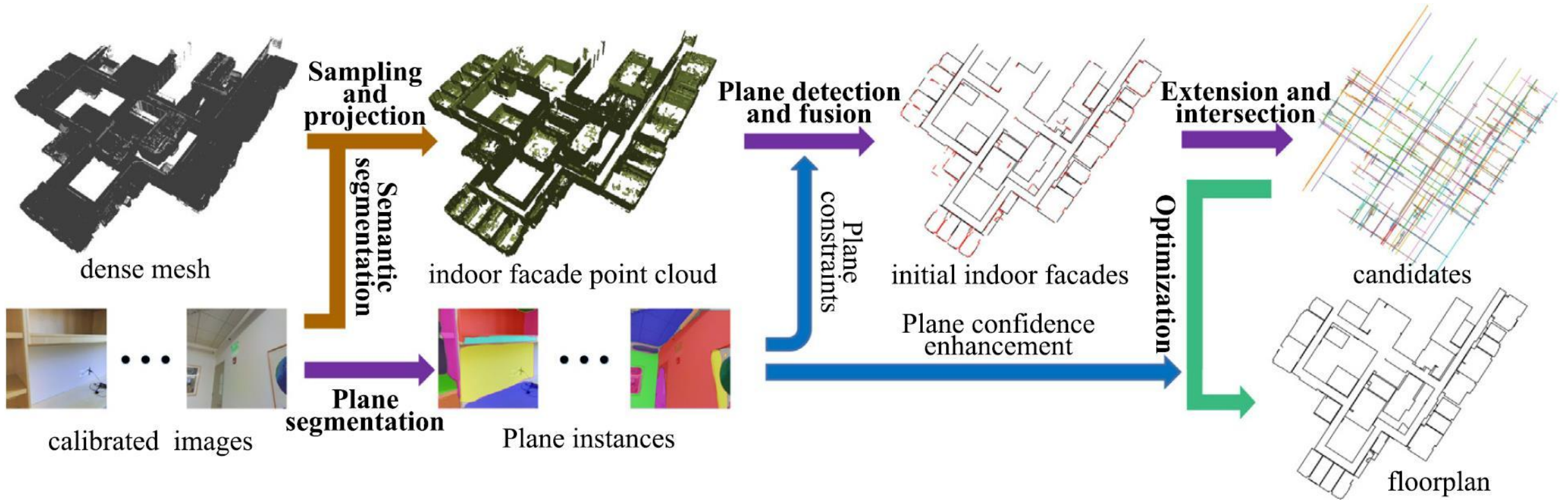
One More Thing

- **VecIM**: Han et al. ISPRS P&RS 2021



One More Thing

- **FloorUSG**: Han et al. ISPRS P&RS 2023



总结展望

• 总结

- 增量式旋转平均: IRA, IJCV 2021
- 层级增量式旋转平均: IRA++, IEEE T-CSVT 2022
- 在线层级增量式旋转平均: IRAv3, IEEE T-CSVT 2023
- 基准层级增量式旋转平均: IRAv3+, IEEE T-CSVT Major Revision
- 增量式平移平均: ITA, IEEE T-CSVT 2022

• 展望

- 在线/基准层级增量式旋转平均 \Rightarrow **在线**基准层级增量式旋转平均
- 增量式平移平均 \Rightarrow 增量式**尺度**平均 + **尺度已知**的增量式平移平均
- 增量式尺度/旋转/平移平均 \Rightarrow 增量式**运动**平均

相关成果

- [1] **X. Gao**, L. Zhu, Z. Xie, H. Liu*, and S. Shen*. [Incremental Rotation Averaging](#)[J]. *IJCV*, 2021.
- [2] **X. Gao**, L. Zhu, H. Cui, Z. Xie, and S. Shen*. [IRA++: Distributed Incremental Rotation Averaging](#)[J]. *IEEE T-CSVT*, 2022.
- [3] **X. Gao**, H. Cui, M. Li, Z. Xie, and S. Shen*. [IRAv3: Hierarchical Incremental Rotation Averaging on the Fly](#)[J]. *IEEE T-CSVT*, 2023.
- [4] **X. Gao**, H. Cui, W. Huang, M. Li, and S. Shen*. [IRAv3+: Hierarchical Incremental Rotation Averaging via Multiple Connected Dominating Sets](#)[J]. *IEEE T-CSVT*, Major Revision.
- [5] **X. Gao**, L. Zhu, B. Fan, H. Liu*, and S. Shen*. [Incremental Translation Averaging](#)[J]. *IEEE T-CSVT*, 2022.
- [6] **X. Gao**, J. Luo, K. Li*, and Z. Xie. [Hierarchical RANSAC-Based Rotation Averaging](#)[J]. *IEEE SPL*, 2020.
- [7] **X. Gao**, S. Shen*, Y. Zhou, H. Cui, L. Zhu, and Z. Hu. [Ancient Chinese Architecture 3D Preservation by Merging Ground and Aerial Point Clouds](#)[J]. *ISPRS P&RS*, 2018.
- [8] H. Cui*, S. Shen, W. Gao, L. Hong*, and Z. Wang. [Efficient and Robust Large-Scale Structure-from-Motion via Track Selection and Camera Prioritization](#)[J]. *ISPRS P&RS*, 2019.
- [9] J. Han, M. Rong, H. Jiang, H. Liu*, and S. Shen*. [Vectorized Indoor Surface Reconstruction from 3D Point Cloud with Multistep 2D Optimization](#)[J]. *ISPRS P&RS*, 2021.
- [10] J. Han#, Y. Liu#, M. Rong, X. Zheng, and S. Shen*. [FloorUSG: Indoor Floorplan Reconstruction by Unifying 2D Semantics and 3D Geometry](#)[J]. *ISPRS P&RS*, 2023.
- [11] J. Han#, L. Zhu#, **X. Gao**, Z. Hu, L. Zhou, H. Liu*, and S. Shen*. [Urban Scene LOD Vectorized Modeling from Photogrammetry Meshes](#)[J]. *IEEE T-IP*, 2021.
- [12] L. Zhu, S. Shen*, **X. Gao**, and Z. Hu. [Large Scale Urban Scene Modeling from MVS Meshes](#)[C]. *In Proc. ECCV*, 2018.

谢 谢

高 翔

中国科学院自动化研究所

电子邮箱: xiang.gao@ia.ac.cn

个人主页: <https://xgao-3dv.github.io/>

