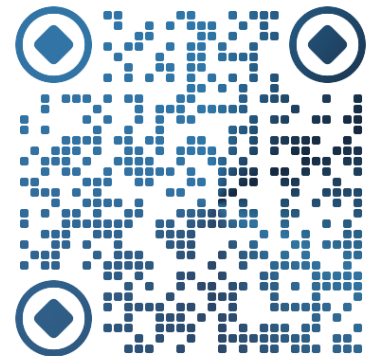


A Brief Introduction to 3D Vision

Xiang Gao, Lecturer

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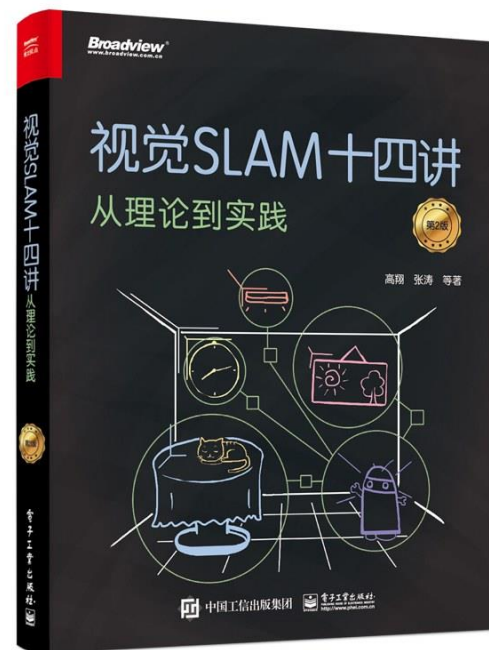
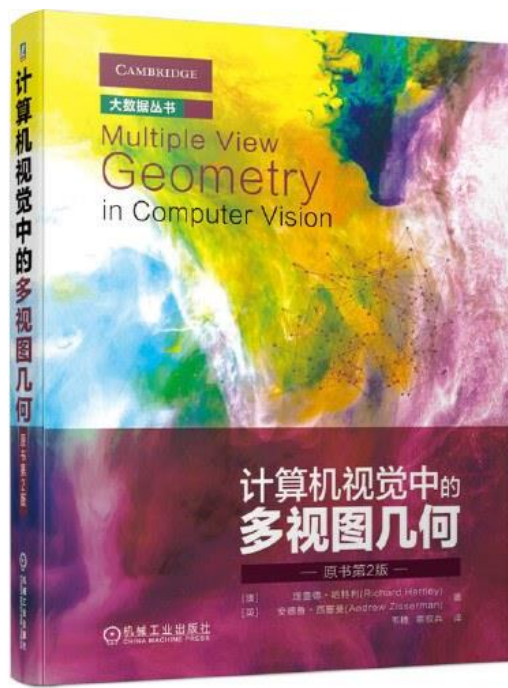
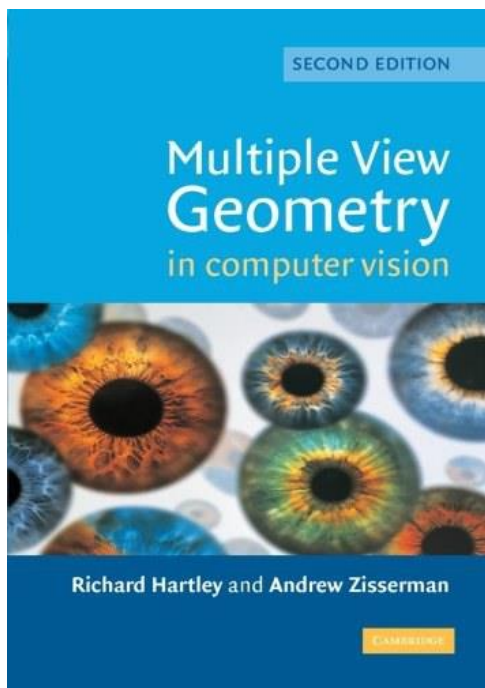


课程目录

- 主要任务
- 相机模型
- 相机标定
- 三角测量
- 位姿估计
- 稀疏重建
- 常用工具

参考书目

- 计算机视觉中的多视图几何：原书第2版/（澳）理查德·哈特利（Richard Hartley），（英）安德鲁·西塞曼（Andrew Zisserman）著；韦穗，章权兵译——北京：机械工业出版社，2020年1月第1版
- 视觉SLAM十四讲：从理论到实践/高翔，张涛等著——北京：电子工业出版社，2019年8月第2版





主要任务

- 通过**传感器 (Sensors)**获取**数据 (Data)**
- **重建 (Reconstruct)**场景的三维结构
- 实现对三维场景的**理解 (Understanding)**

主要任务

- 传感器
 - 数码相机 (Digital Camera)



Canon EOS 5D Mark IV



Apple iPhone 12



DJI Mini 2

主要任务

- 传感器
 - 深度相机 (RGB-D Camera)



Microsoft Kinect



Intel Realsense



Apple iPhone 12 Pro

主要任务

- 传感器
 - 激光扫描仪 (Laser Scanner)



Leica ScanStation P30



FARO Focus 3D X330

主要任务

- 传感器

- 激光雷达 (Light Detection And Ranging, LiDAR)



Velodyne HDL-32E



Velodyne HDL-64E

主要任务

- 数据
 - 彩色图像



Ground image examples of FGT



Ground image examples of NCT



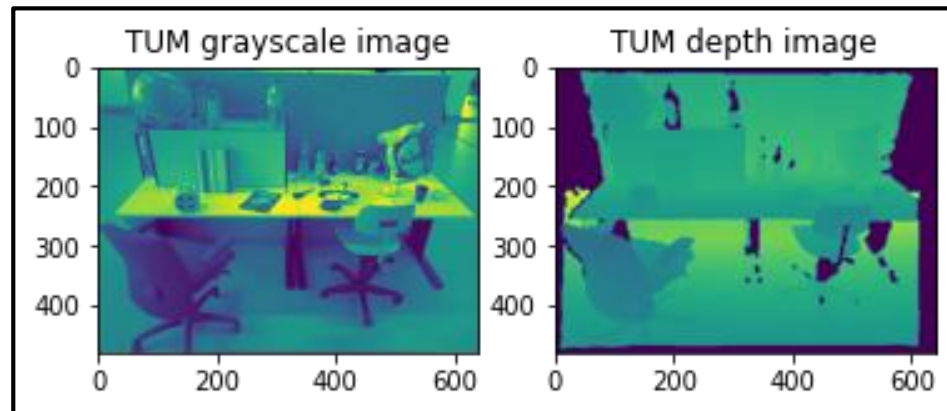
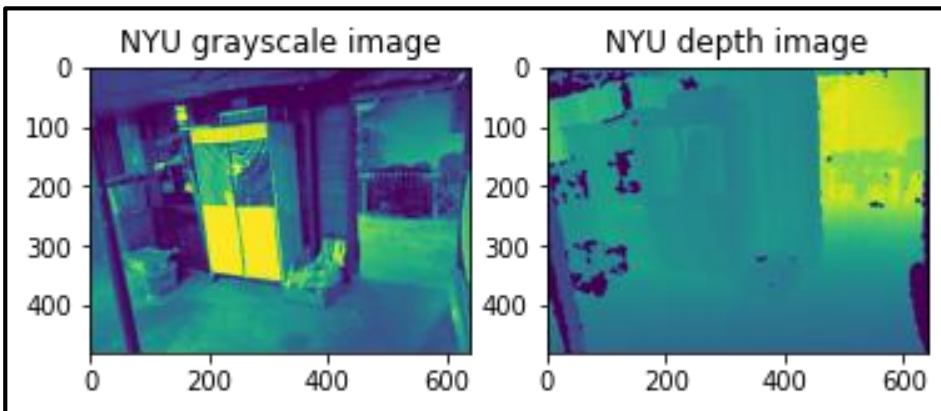
Aerial image examples of FGT



Aerial image examples of NCT

主要任务

- 数据
 - 深度图像



主要任务

- 数据
 - 激光点云——激光扫描仪



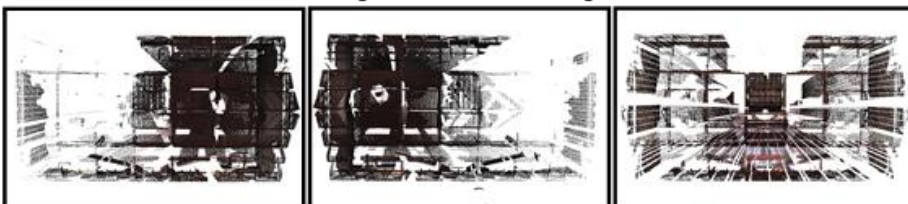
Outdoor laser point cloud examples of FGT



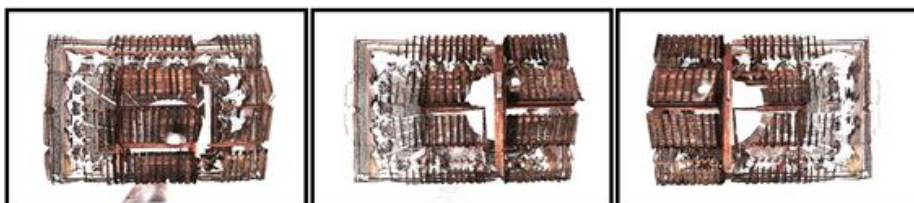
Outdoor laser point cloud examples of NCT



Indoor laser point cloud examples of GEH



Indoor laser point cloud examples of MJH



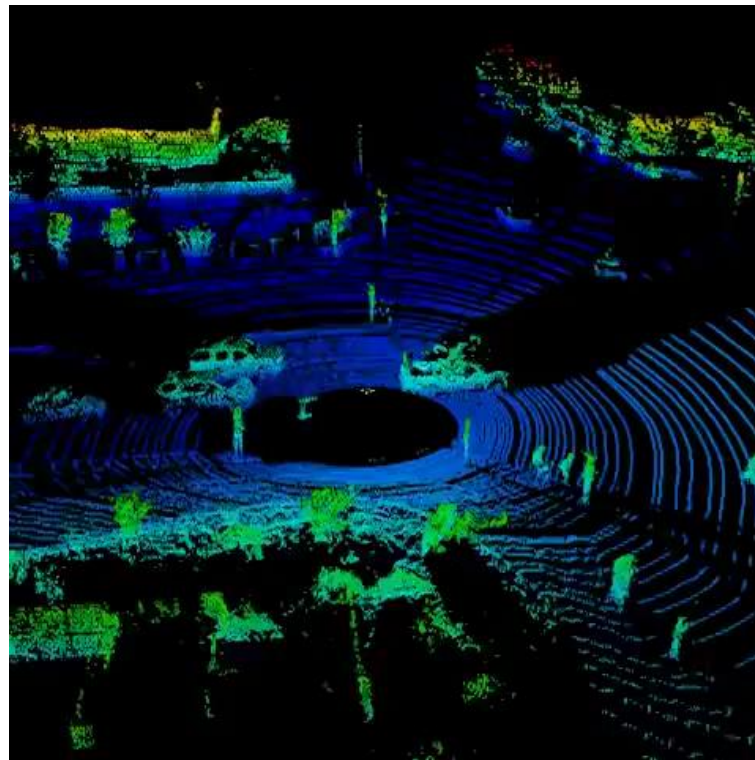
Indoor laser point cloud examples of GRH



Indoor laser point cloud examples of GBH

主要任务

- 数据
 - 激光点云——激光雷达



主要任务

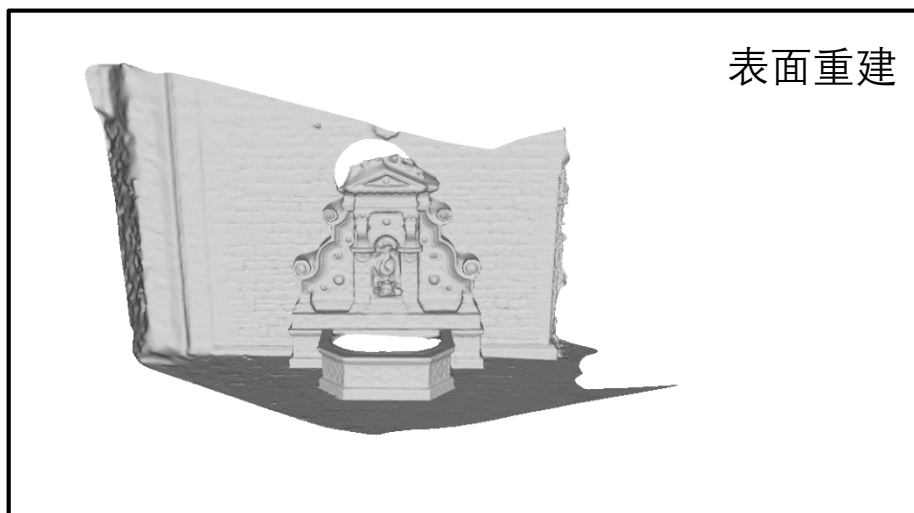
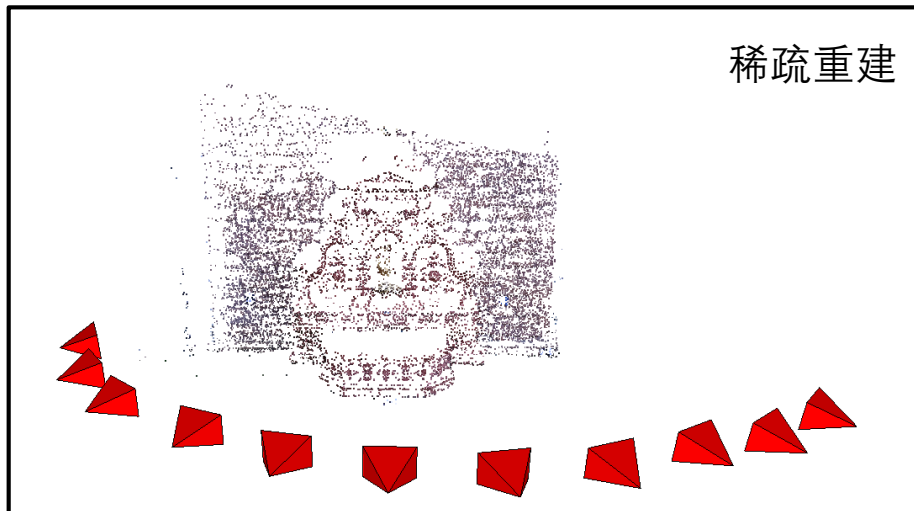
- 数据
 - 数据比较

	彩色图像	深度图像	激光点云 激光扫描仪	激光点云 激光雷达
纹理信息	😊😊	😊	😊	😞
深度信息	😞	😊	😊	😊
数据精度	😞	😞	😊😊	😊
测量范围	😊	😞	😊😊	😊
噪声程度	😞	😞	😊😊	😊
设备成本	😊	😊	😞	😞
环境适应性	😞	😞	😊	😊
信息丰富度	😊	😊	😊	😞
空间连续性	😊	😊	😊	😞
操作灵活性	😊😊	😊	😞	😊

主要任务

- 重建

- 数码相机——基于图像的建模 (Image-Based Modeling)



主要任务

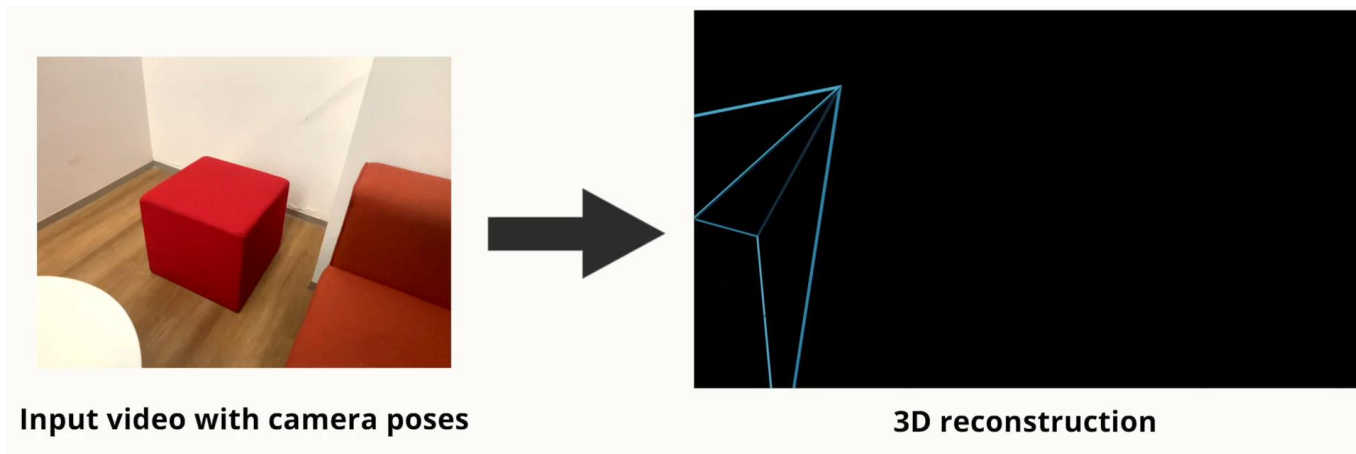
- 重建
 - 数码相机——融合航拍与地面图像的室外场景重建



主要任务

- 重建

- 数码相机——基于单目视频的实时场景重建



主要任务

- 重建

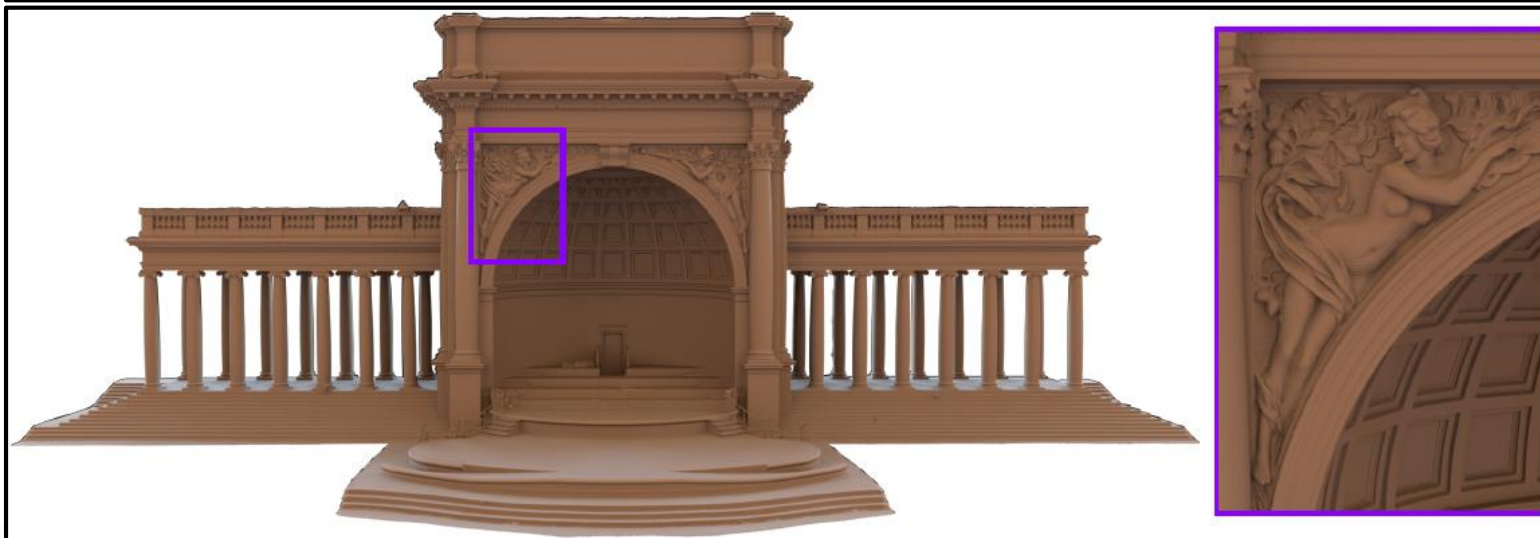
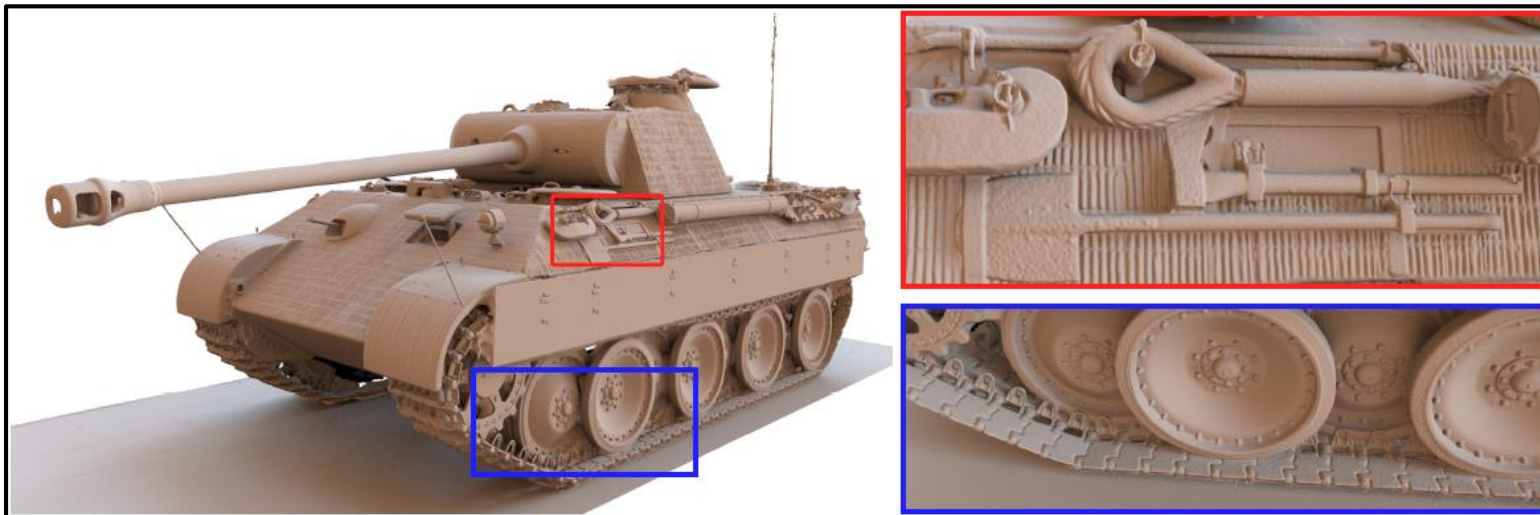
- 深度相机——基于RGB-D图像的室内场景重建



主要任务

- 重建

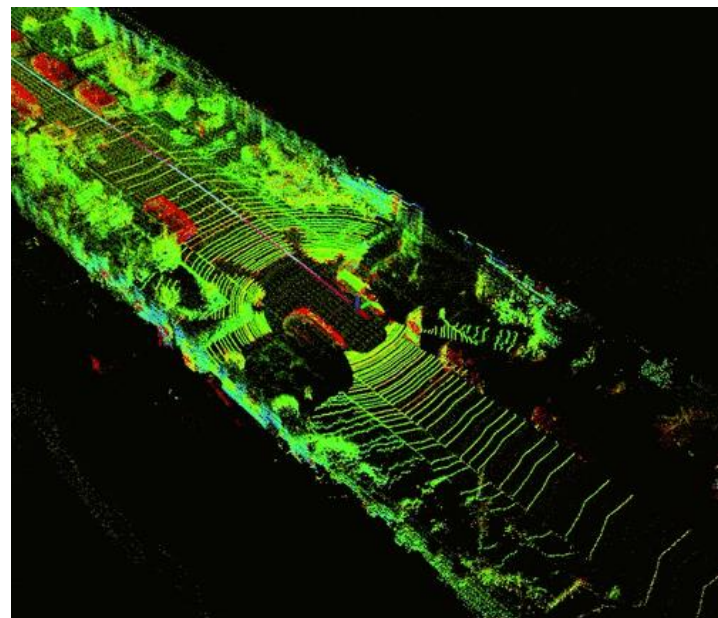
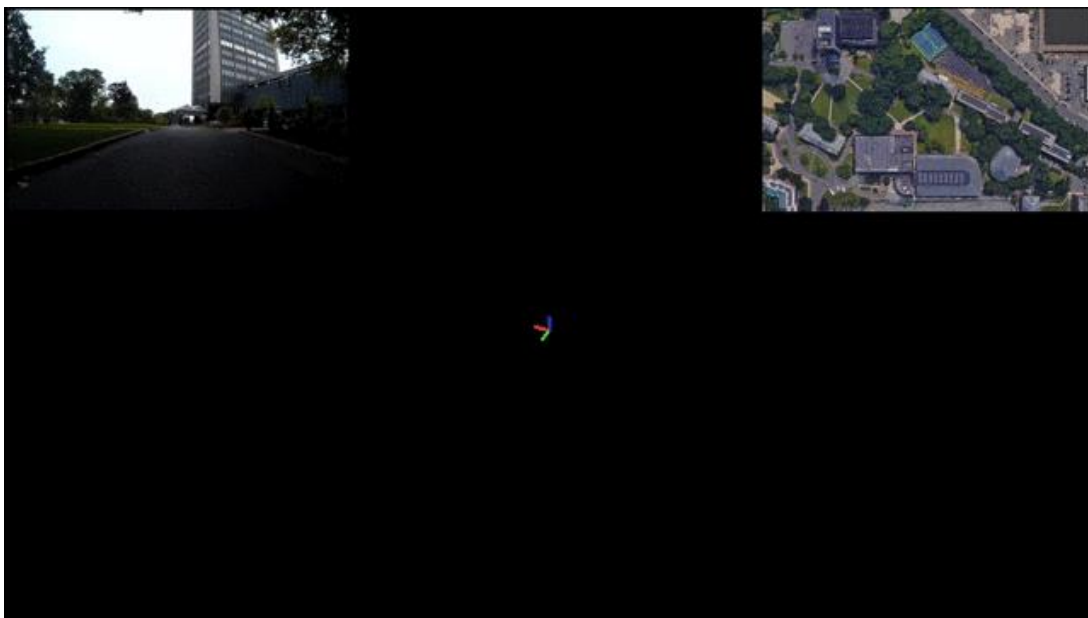
- 激光扫描仪——基于激光点云对齐的场景重建



主要任务

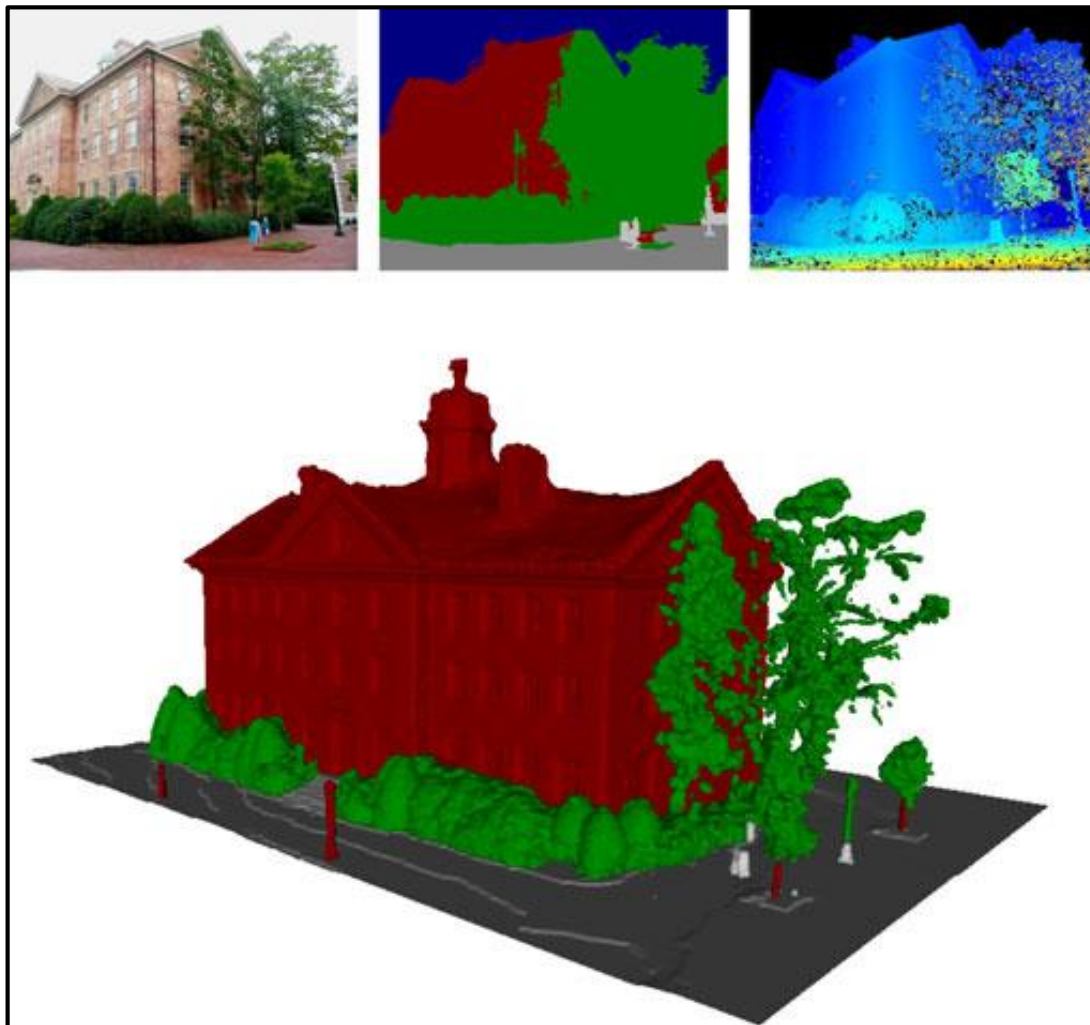
- 重建

- 激光雷达——基于LOAM的场景重建
 - LOAM: LiDAR Odometry And Mapping



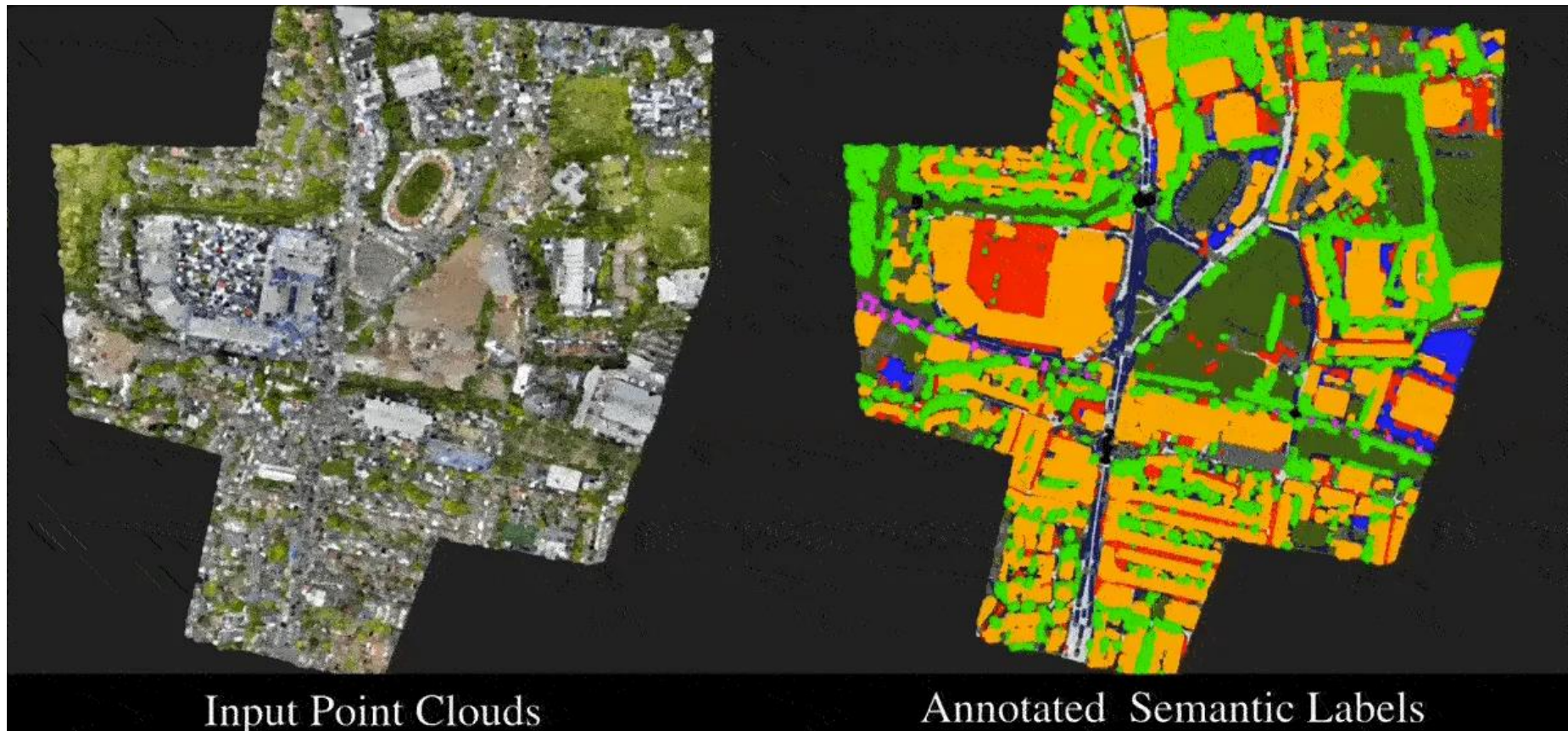
主要任务

- 理解
 - 语义分割 (Semantic Segmentation)



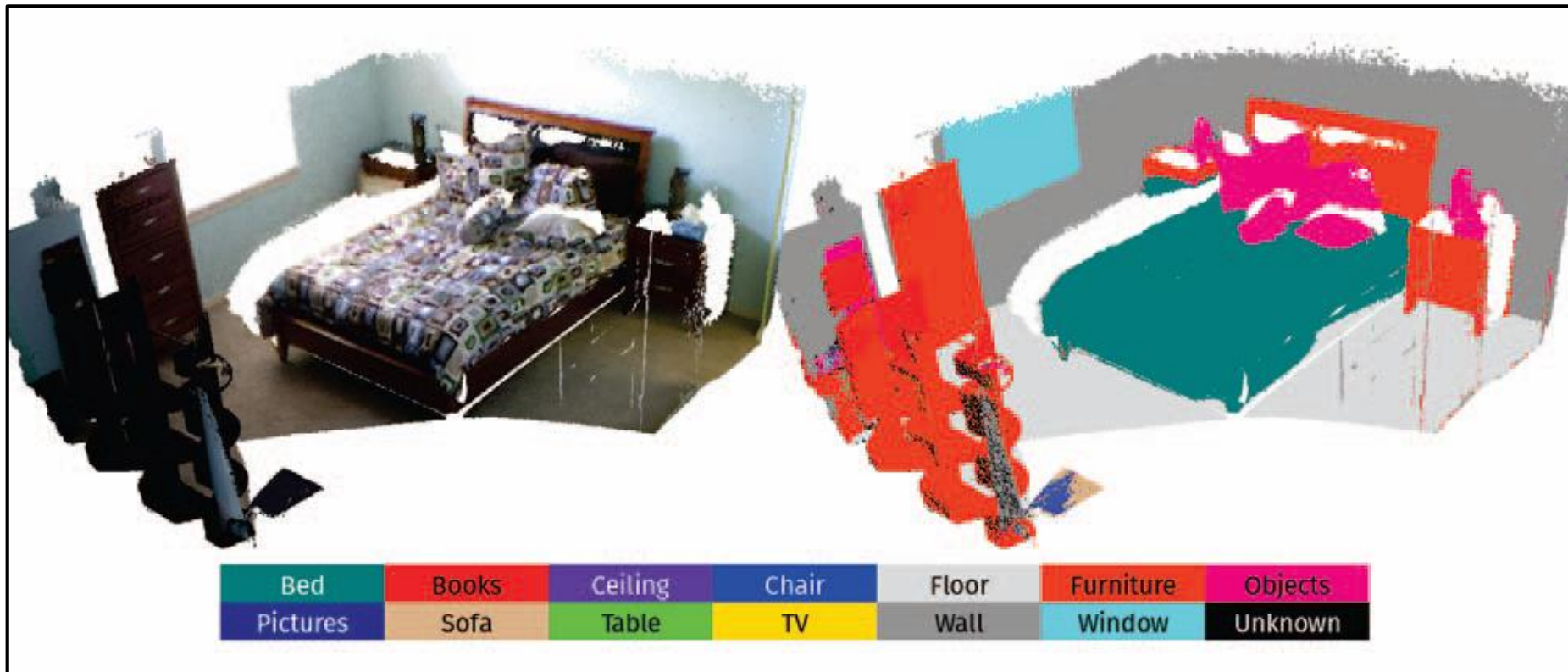
主要任务

- 理解
 - 语义分割 (Semantic Segmentation)



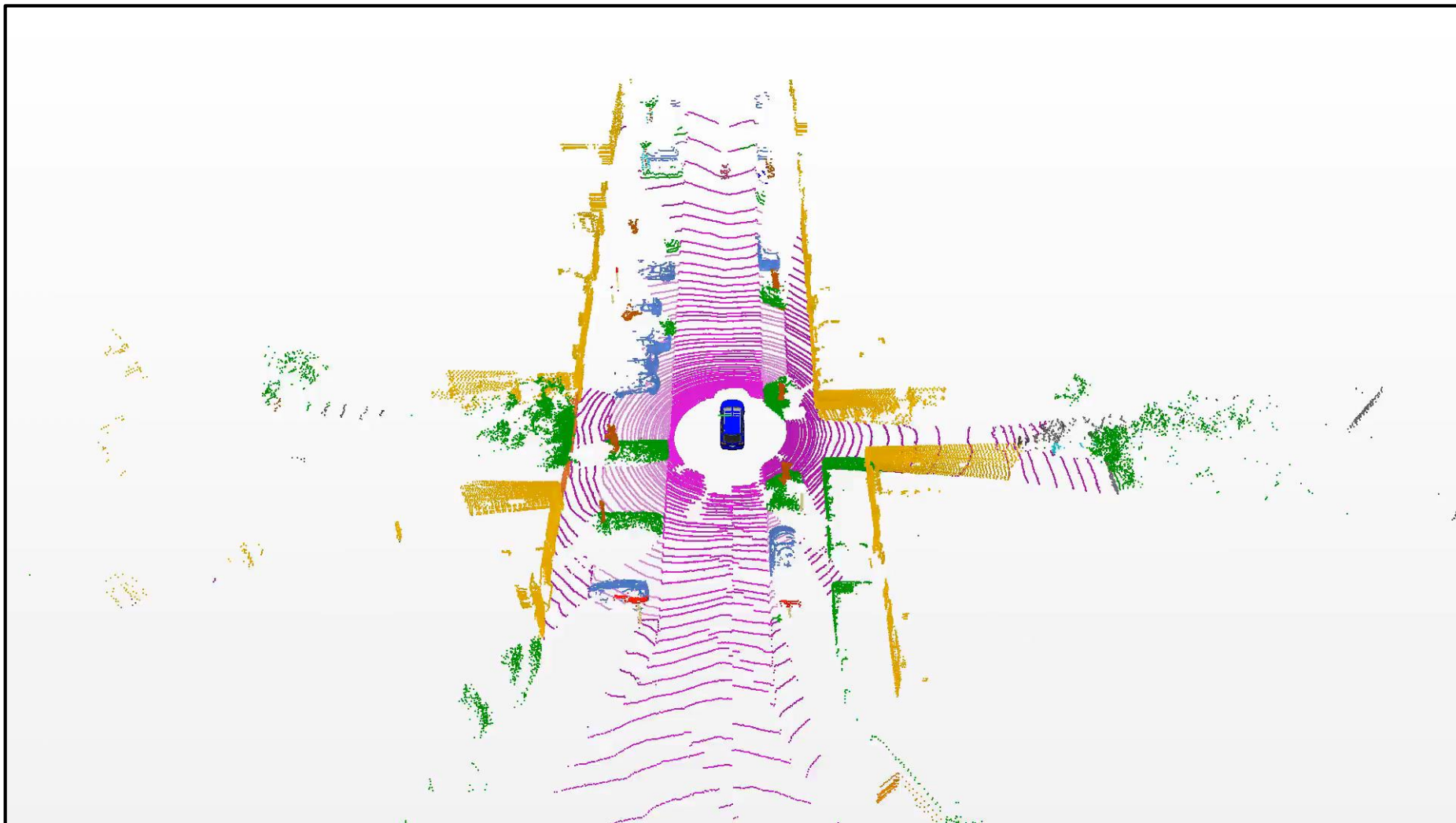
主要任务

- 理解
 - 语义分割 (Semantic Segmentation)



主要任务

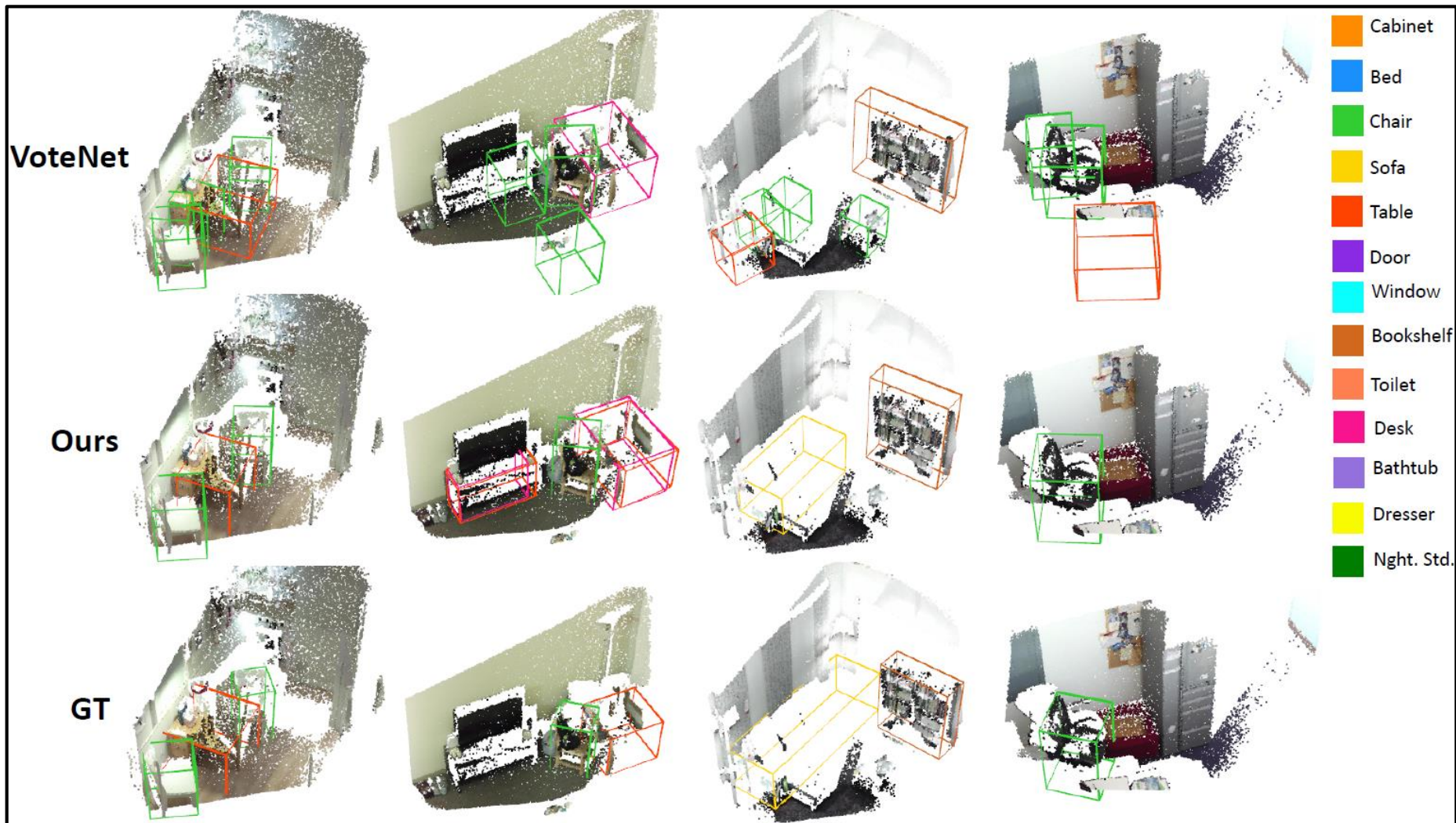
- 理解
 - 语义分割 (Semantic Segmentation)



主要任务

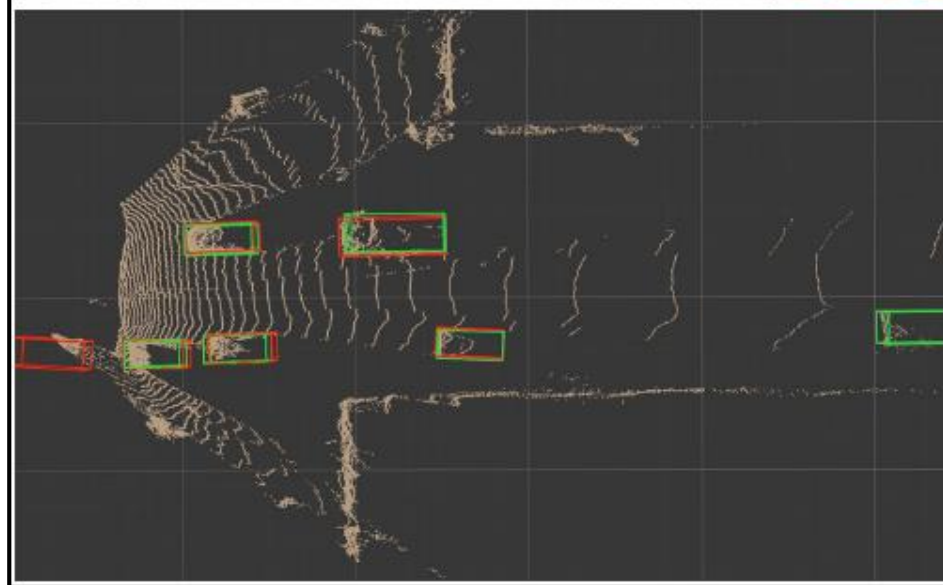
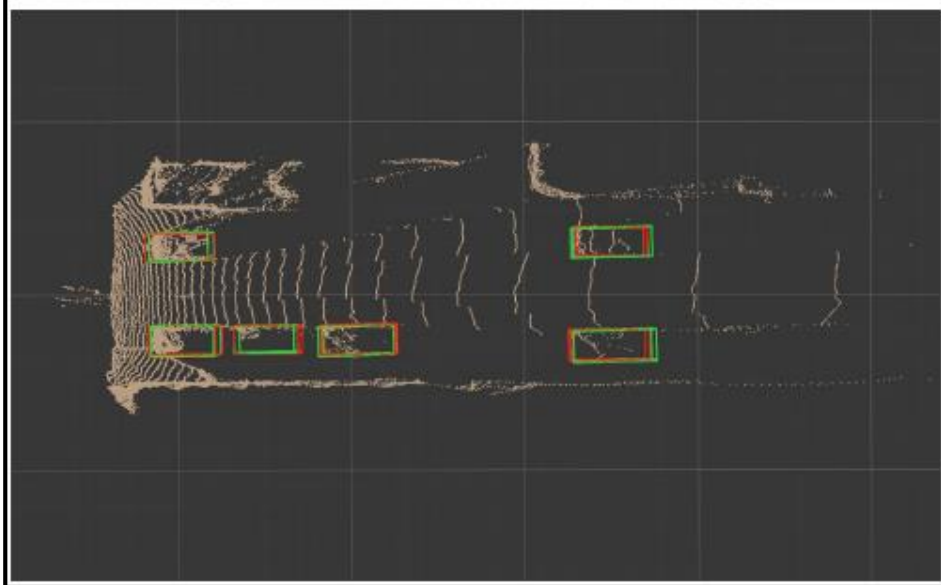
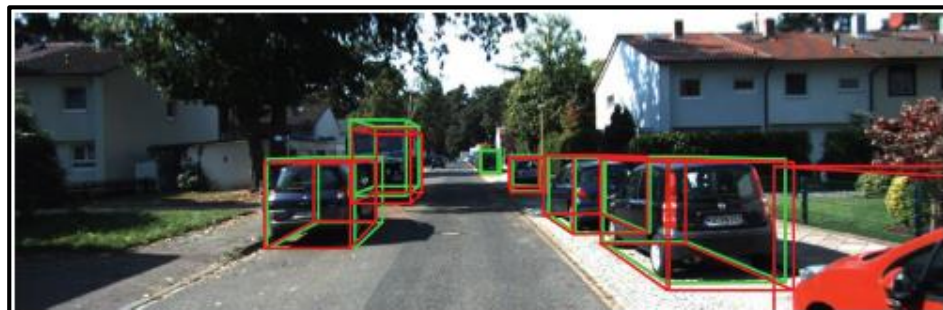
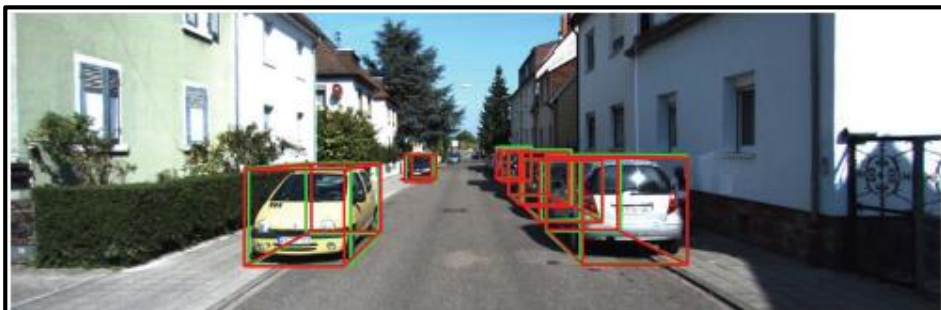
- 理解

- 目标检测 (Object Detection)



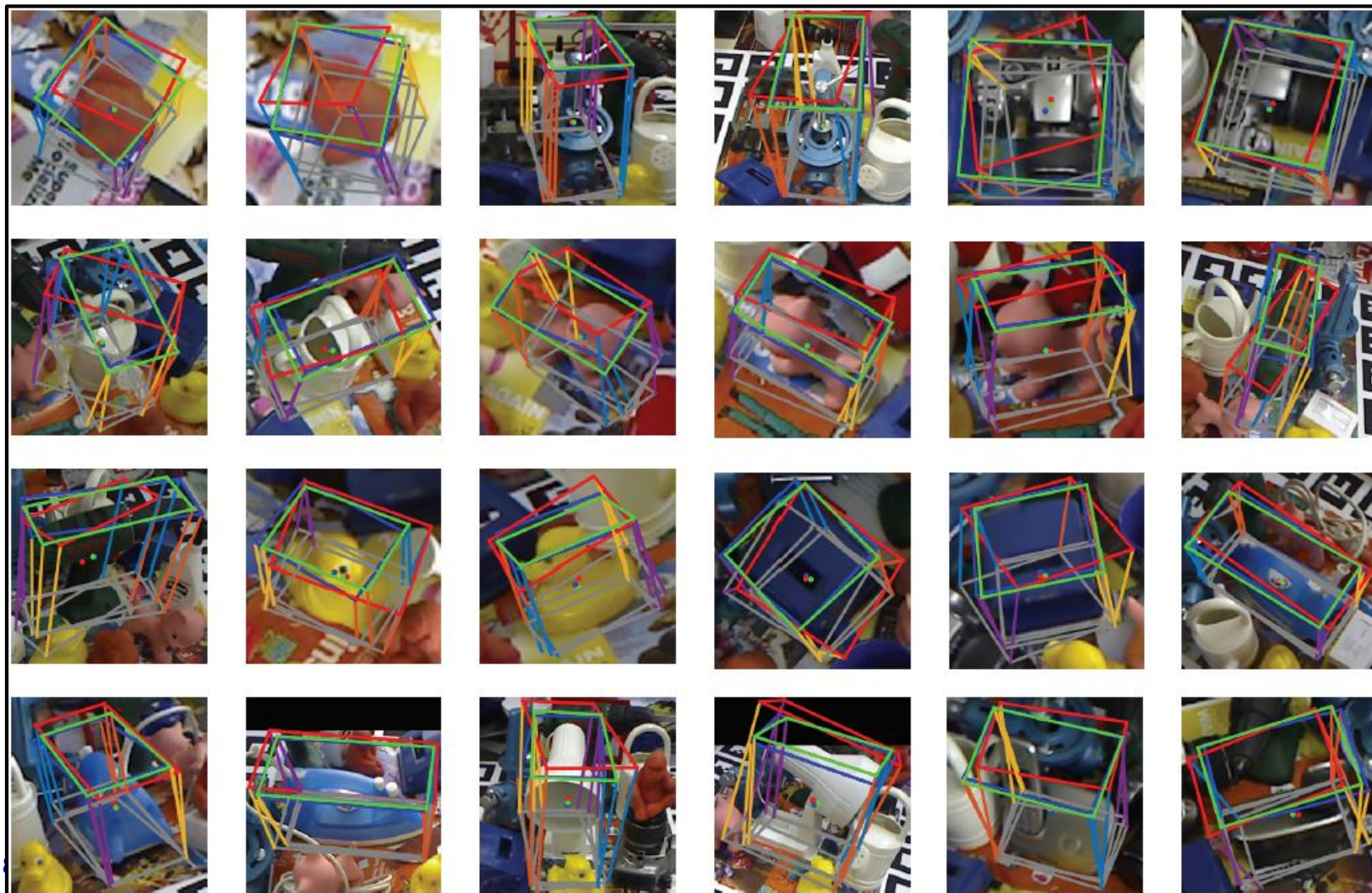
主要任务

- 理解
 - 目标检测 (Object Detection)



主要任务

- 理解
 - 位姿估计 (Pose Estimation)



相机模型

- 基本概念

- 用于描述从三维空间到二维图像映射关系的数学模型

- 符号表示

- 标量：小写/大写斜体(x, X); 向量：小写/大写粗体(\mathbf{x}, \mathbf{X}); 矩阵：大写正体(P)

- 齐次坐标 (Homogeneous Coordinate)

- $$\begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

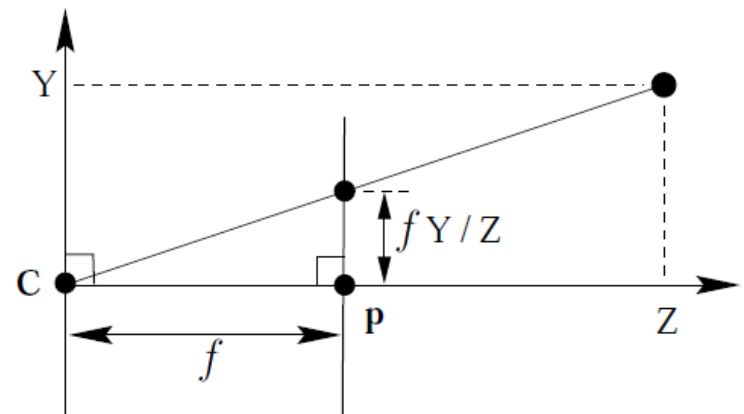
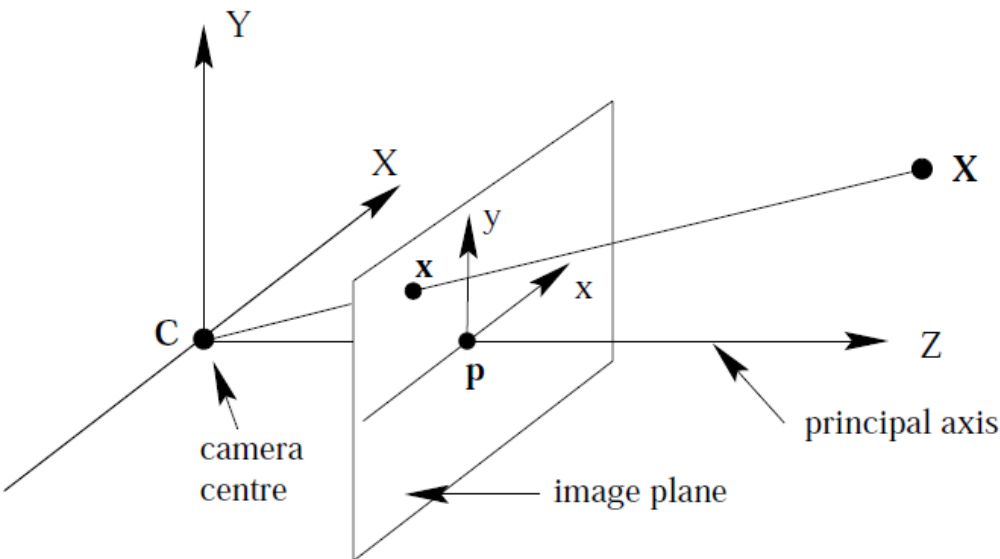
- $$\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \Leftrightarrow \begin{pmatrix} \omega x \\ \omega y \\ \omega \end{pmatrix} = \omega \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

- 引入齐次坐标的意义

- 引入无穷远概念
- 便于矩阵运算

相机模型

- 基本针孔模型 (Pinhole Model)



- 光心 \mathbf{C} (Camera Centre), 主轴 $\overline{\mathbf{CZ}}$ (Principal Axis), 主点 \mathbf{p} (Principal Point), 焦距 $f = \|\mathbf{p} - \mathbf{C}\|$ (Focal Length)

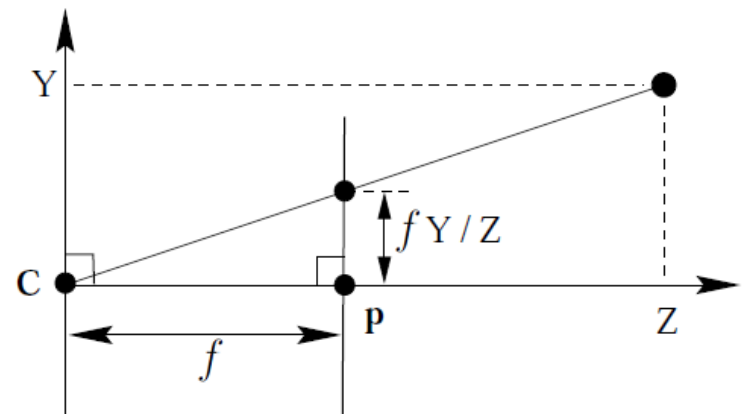
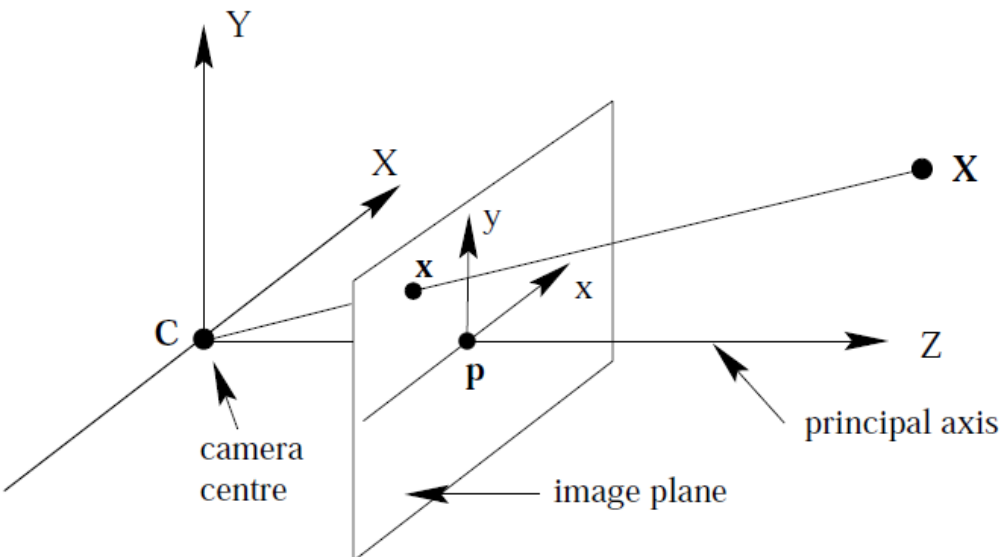
- 图像平面 $Z = f$ (Image Plane)

- 空间点 $\mathbf{X} = (X, Y, Z)^T$, 图像点 $\mathbf{x} = (x, y)^T$

- $$\frac{x}{X} = \frac{y}{Y} = \frac{f}{Z} \Rightarrow \begin{cases} x = fX/Z \\ y = fY/Z \end{cases}$$

相机模型

- 用齐次坐标表示中心投影

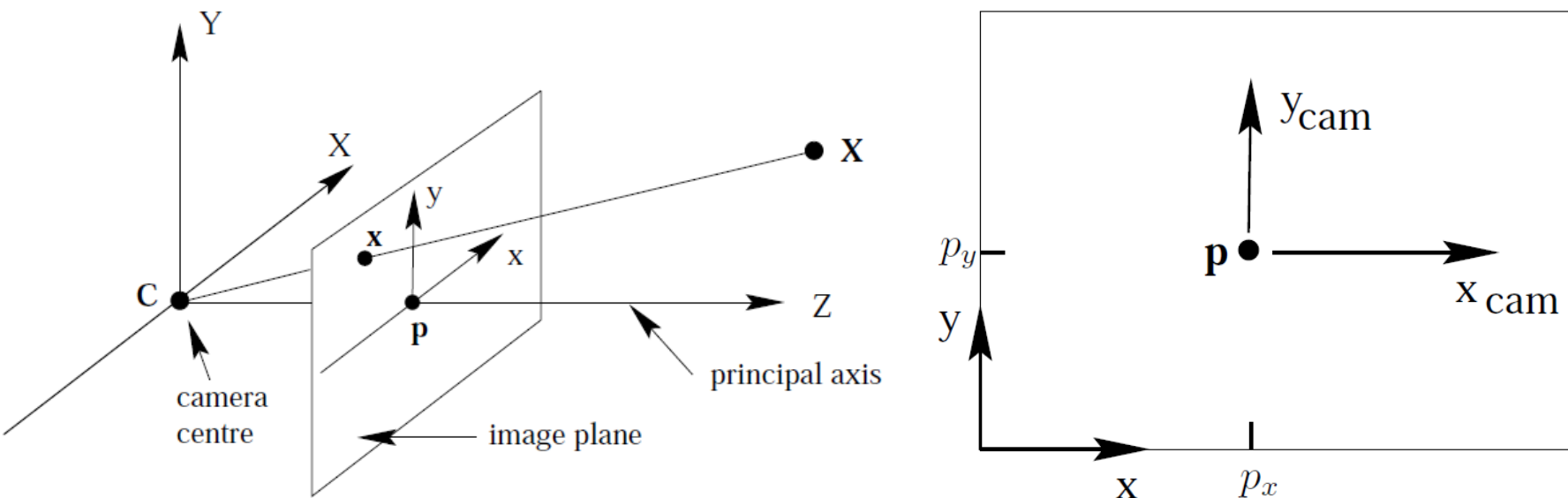


- $$\begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \mapsto \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \Leftrightarrow Z \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = Z \begin{pmatrix} fX/Z \\ fY/Z \\ 1 \end{pmatrix} = \begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

- $$\begin{cases} \mathbf{X} \stackrel{\text{def}}{=} (X, Y, Z, 1)^T \\ \mathbf{x} \stackrel{\text{def}}{=} (fX/Z, fY/Z, 1)^T \Rightarrow Z\mathbf{x} = P\mathbf{X}, \text{ 矩阵 } P \text{ 称为投影 (Projection) 矩阵} \\ P \stackrel{\text{def}}{=} \text{diag}(f, f, 1) [\mathbf{I} | \mathbf{0}] \end{cases}$$

相机模型

• 主点偏置



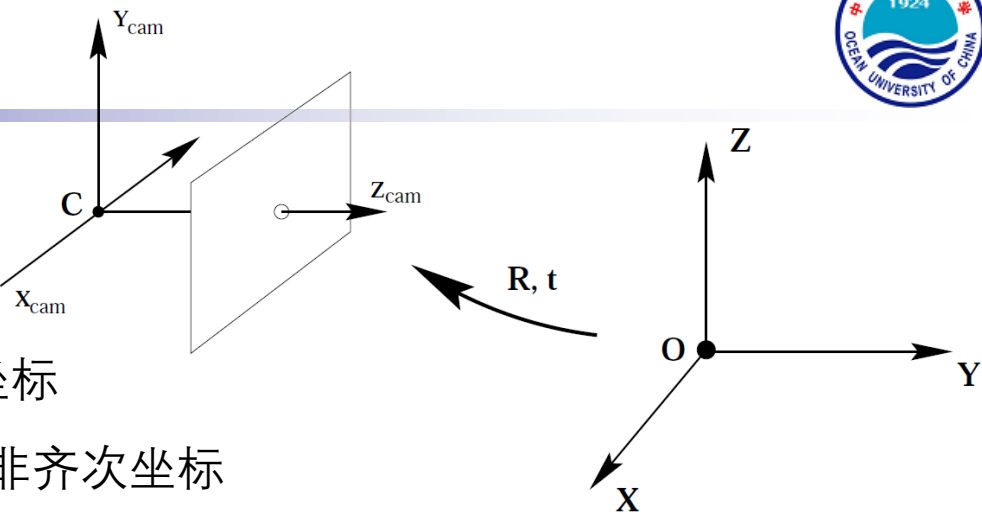
- 若在成像平面左下角建立坐标系，且将主点坐标记为 $\mathbf{p} = (p_x, p_y)^T$ ：

$$\bullet \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \mapsto \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \Leftrightarrow Z \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = Z \begin{pmatrix} x_{cam} + p_x \\ y_{cam} + p_y \\ 1 \end{pmatrix} = \begin{pmatrix} fX + Zp_x \\ fY + Zp_y \\ Z \end{pmatrix} = \begin{bmatrix} f & 0 & p_x & 0 \\ 0 & f & p_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

- 记 $K = \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow Z_{cam} \mathbf{x} = K[\mathbf{I}|\mathbf{0}]\mathbf{X}_{cam}$ ，矩阵 K 称为标定 (Calibration) 矩阵

相机模型

• 平移与旋转



- $\tilde{\mathbf{X}}$: 世界坐标系下的非齐次坐标
- $\tilde{\mathbf{X}}_{cam}$: 相机坐标系下的非齐次坐标
- $\tilde{\mathbf{C}}$: 相机中心在世界坐标系下的非齐次坐标
- 则, $\tilde{\mathbf{X}}_{cam} = R(\tilde{\mathbf{X}} - \tilde{\mathbf{C}})$, 写成齐次形式:
- $\mathbf{X}_{cam} = \begin{bmatrix} R & -R\tilde{\mathbf{C}} \\ \mathbf{0}^T & 1 \end{bmatrix} \begin{pmatrix} \tilde{\mathbf{X}} \\ 1 \end{pmatrix} = \begin{bmatrix} R & -R\tilde{\mathbf{C}} \\ \mathbf{0}^T & 1 \end{bmatrix} \mathbf{X}$
- $Z_{cam}\mathbf{x} = K[I|\mathbf{0}]\mathbf{X}_{cam} \Rightarrow Z_{cam}\mathbf{x} = K[I|\mathbf{0}] \begin{bmatrix} R & -R\tilde{\mathbf{C}} \\ \mathbf{0}^T & 1 \end{bmatrix} \mathbf{X} = KR[I|-\tilde{\mathbf{C}}]\mathbf{X}$
- 记 $\mathbf{t} = -R\tilde{\mathbf{C}}$, 则 $Z_{cam}\mathbf{x} = P\mathbf{X}$, $P = K[R|\mathbf{t}]$

相机模型

- 畸变模型 (Distortion Model)

- 径向畸变 (Radial Distortion)

- $$\begin{cases} x_{r_distorted} = \tilde{x}(1 + k_1r^2 + k_2r^4 + k_3r^6) \\ y_{r_distorted} = \tilde{y}(1 + k_1r^2 + k_2r^4 + k_3r^6) \end{cases}$$

- 切向畸变 (Tangential Distortion)

- $$\begin{cases} x_{t_distorted} = \tilde{x} + 2p_1\tilde{x}\tilde{y} + p_2(r^2 + 2\tilde{x}^2) \\ y_{t_distorted} = \tilde{y} + p_1(r^2 + 2\tilde{y}^2) + 2p_2\tilde{x}\tilde{y} \end{cases}$$

- 径向+切向畸变

- $$\begin{cases} x_{distorted} = \tilde{x}(1 + k_1r^2 + k_2r^4 + k_3r^6) + 2p_1\tilde{x}\tilde{y} + p_2(r^2 + 2\tilde{x}^2) \\ y_{distorted} = \tilde{y}(1 + k_1r^2 + k_2r^4 + k_3r^6) + p_1(r^2 + 2\tilde{y}^2) + 2p_2\tilde{x}\tilde{y} \end{cases}$$

- $$\begin{cases} r^2 = \tilde{x}^2 + \tilde{y}^2 \\ \tilde{x} = (x_{actual} - p_x)/f \\ \tilde{y} = (y_{actual} - p_y)/f \end{cases} \Rightarrow \begin{cases} x_{ideal} = fx_{distorted} + p_x \\ y_{ideal} = fy_{distorted} + p_y \end{cases}$$



相机标定 (Camera Calibration)

- 基本概念
 - 已知若干二维图像点与三维空间点的对应关系及其坐标
 - 求解相机内/外参数
 - $\mathbf{x}_i \leftrightarrow \mathbf{X}_i \Rightarrow K, R, \mathbf{t}$
- 主要方法
 - 基于三维靶标的标定方法
 - 基于二维靶标的标定方法

相机标定

- 基于三维靶标的标定方法

- 直接线性变换 (Direct Linear Transformation, DLT)

- 一对2D-3D对应点可提供如下约束关系:

- $$\lambda_i \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{pmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{pmatrix} \Rightarrow$$

$$\begin{cases} \lambda_i x_i = p_{11}X_i + p_{12}Y_i + p_{13}Z_i + p_{14} \\ \lambda_i y_i = p_{21}X_i + p_{22}Y_i + p_{23}Z_i + p_{24} \\ \lambda_i = p_{31}X_i + p_{32}Y_i + p_{33}Z_i + p_{34} \end{cases} \Rightarrow$$

$$\begin{cases} (p_{31}X_i + p_{32}Y_i + p_{33}Z_i + p_{34})x_i = p_{11}X_i + p_{12}Y_i + p_{13}Z_i + p_{14} \\ (p_{31}X_i + p_{32}Y_i + p_{33}Z_i + p_{34})y_i = p_{21}X_i + p_{22}Y_i + p_{23}Z_i + p_{24} \end{cases}$$

相机标定

- 基于三维靶标的标定方法
 - 上式写成矩阵相乘的形式:

$$\bullet \begin{bmatrix} X_i & Y_i & Z_i & 1 & 0 & 0 & 0 & 0 & -x_i X_i & -x_i Y_i & -x_i Z_i & -x_i \\ 0 & 0 & 0 & 0 & X_i & Y_i & Z_i & 1 & -y_i X_i & -y_i Y_i & -y_i Z_i & -y_i \end{bmatrix} \begin{pmatrix} p_{11} \\ p_{12} \\ p_{13} \\ p_{14} \\ p_{21} \\ p_{22} \\ p_{23} \\ p_{24} \\ p_{31} \\ p_{32} \\ p_{33} \\ p_{34} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

相机标定

- 基于三维靶标的标定方法

- 将 $n(n \geq 6)$ 对2D-3D对应点提供的约束关系堆叠在一起，可得：

$$\bullet \begin{bmatrix} X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -x_1 X_1 & -x_1 Y_1 & -x_1 Z_1 & -x_1 \\ 0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -y_1 X_1 & -y_1 Y_1 & -y_1 Z_1 & -y_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ X_n & Y_n & Z_n & 1 & 0 & 0 & 0 & 0 & -x_n X_n & -x_n Y_n & -x_n Z_n & -x_n \\ 0 & 0 & 0 & 0 & X_n & Y_n & Z_n & 1 & -y_n X_n & -y_n Y_n & -y_n Z_n & -y_n \end{bmatrix} \begin{pmatrix} p_{11} \\ p_{12} \\ p_{13} \\ p_{14} \\ p_{21} \\ p_{22} \\ p_{23} \\ p_{24} \\ p_{31} \\ p_{32} \\ p_{33} \\ p_{34} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix}$$

- 写成矩阵运算形式： $A(2n \times 12)\mathbf{p}(12 \times 1) = \mathbf{0}(2n \times 1)$
- 对矩阵A进行奇异值分解 (Singular Value Decomposition, SVD)，对应于最小奇异值的奇异向量为 $A\mathbf{p} = \mathbf{0}$ 的最小二乘解 $\mathbf{p}_{\text{algebraic}}$
- 若 $A = UDV^T$ ，则 $\mathbf{p}_{\text{algebraic}}$ 为V的最后一列

相机标定

- 基于三维靶标的标定方法

- 代数误差与几何误差

- 代数误差: $\mathbf{p}_{\text{algebraic}} = \operatorname{argmin} \|\mathbf{A}\mathbf{p}\|_2$

- 几何误差: $P_{\text{geometric}} = \operatorname{argmin} \sum_{i=1}^n \|\mathbf{x}_i - \mathbf{P}\mathbf{X}_i\|_2$

- 因上述问题为**非线性最小二乘问题**，需要采用迭代的方式进行优化求解，并采用 $\mathbf{p}_{\text{algebraic}}$ 对P进行初始化



相机标定

- 基于三维靶标的标定方法

- 投影矩阵P的分解

- $P = KR[I|R^{-1}\mathbf{t}] = [KR|K\mathbf{t}] = [M|\mathbf{p}_4]$

- 对矩阵M进行RQ分解， $M = RQ$ ，R为（右）上三角矩阵，对应标定矩阵K，为Q正交矩阵，对应旋转矩阵R

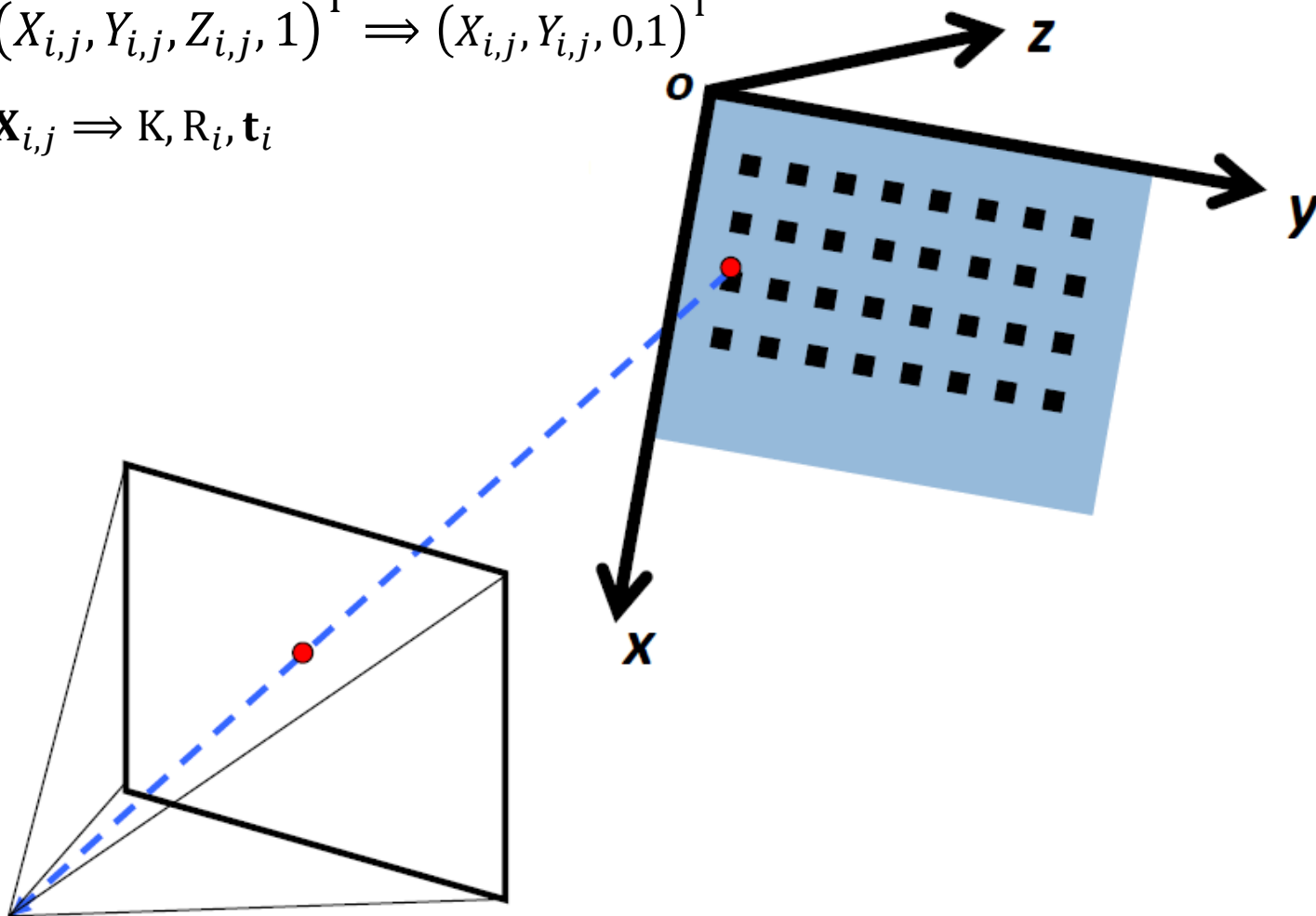
- $\mathbf{t} = K^{-1}\mathbf{p}_4$

相机标定

- 基于二维靶标的标定方法

- $$\mathbf{X}_{i,j} = (X_{i,j}, Y_{i,j}, Z_{i,j}, 1)^T \Rightarrow (X_{i,j}, Y_{i,j}, 0, 1)^T$$

- $$\mathbf{x}_{i,j} \leftrightarrow \mathbf{X}_{i,j} \Rightarrow K, R_i, \mathbf{t}_i$$



Z. Zhang. *A Flexible New Technique for Camera Calibration*. *IEEE Transactions on Pattern Analysis and Machine Intelligence (T-PAMI)*, 2000, 22(11): 1330-1334. **16k+ Citations**

相机标定

- 基于二维靶标的标定方法

- 一对二维靶标上的空间点与其对应的图像点可提供如下约束关系：

- $$\lambda_{i,j} \begin{pmatrix} x_{i,j} \\ y_{i,j} \\ 1 \end{pmatrix} = K[R_i | \mathbf{t}_i] \begin{pmatrix} X_{i,j} \\ Y_{i,j} \\ 0 \\ 1 \end{pmatrix} = K[\mathbf{r}_1^i \quad \mathbf{r}_2^i \quad \mathbf{r}_3^i \quad \mathbf{t}_i] \begin{pmatrix} X_{i,j} \\ Y_{i,j} \\ 0 \\ 1 \end{pmatrix} =$$

$$\underbrace{K[\mathbf{r}_1^i \quad \mathbf{r}_2^i \quad \mathbf{t}_i]}_{H_i} \begin{pmatrix} X_{i,j} \\ Y_{i,j} \\ 1 \end{pmatrix} = \begin{bmatrix} h_{11}^i & h_{12}^i & h_{13}^i \\ h_{21}^i & h_{22}^i & h_{23}^i \\ h_{31}^i & h_{32}^i & h_{33}^i \end{bmatrix} \begin{pmatrix} X_{i,j} \\ Y_{i,j} \\ 1 \end{pmatrix}$$

- 矩阵 H_i 称为单应 (Homography) 矩阵
- 给定 n 对对应点 $\mathbf{x}_{i,j} \leftrightarrow \mathbf{X}_{i,j}$ ， H_i 可通过DLT算法进行估计

相机标定

- 基于二维靶标的标定方法

- 由 $\{H_i\}$ 估计K:

- $H_i = [\mathbf{h}_1^i \quad \mathbf{h}_2^i \quad \mathbf{h}_3^i] = K[\mathbf{r}_1^i \quad \mathbf{r}_2^i \quad \mathbf{t}_i] \Rightarrow K^{-1}[\mathbf{h}_1^i \quad \mathbf{h}_2^i \quad \mathbf{h}_3^i] = [\mathbf{r}_1^i \quad \mathbf{r}_2^i \quad \mathbf{t}_i]$

- 由正交矩阵的性质:
$$\begin{cases} (\mathbf{r}_1^i)^T \mathbf{r}_2^i = 0 \\ \|\mathbf{r}_1^i\| = \|\mathbf{r}_2^i\| = 1 \end{cases}$$

- 可得:
$$\begin{cases} (\mathbf{h}_1^i)^T K^{-T} K^{-1} \mathbf{h}_2^i = 0 \\ (\mathbf{h}_1^i)^T K^{-T} K^{-1} \mathbf{h}_1^i = (\mathbf{h}_2^i)^T K^{-T} K^{-1} \mathbf{h}_2^i \end{cases}$$

- 每幅图像可以获得2个关于K的约束方程
 - K上三角矩阵 $\Rightarrow \omega = K^{-T}K^{-1}$ 为对称矩阵
 - 当图像数目 ≥ 3 时, 可以通过DLT线性解出 ω
 - 进而通过Cholesky分解求出K

相机标定

- 基于二维靶标的标定方法

- 由 H_i 估计 R_i, \mathbf{t}_i :

- $H_i = [\mathbf{h}_1^i \quad \mathbf{h}_2^i \quad \mathbf{h}_3^i] = K[\mathbf{r}_1^i \quad \mathbf{r}_2^i \quad \mathbf{t}_i] \Rightarrow K^{-1}[\mathbf{h}_1^i \quad \mathbf{h}_2^i \quad \mathbf{h}_3^i] = [\mathbf{r}_1^i \quad \mathbf{r}_2^i \quad \mathbf{t}_i]$

- 可得:
$$\begin{cases} \mathbf{r}_1^i = K^{-1}\mathbf{h}_1^i \\ \mathbf{r}_2^i = K^{-1}\mathbf{h}_2^i \\ \mathbf{t}_i = K^{-1}\mathbf{h}_3^i \end{cases}$$

- 由正交矩阵的性质: $\mathbf{r}_3^i = \mathbf{r}_1^i \times \mathbf{r}_2^i = (K^{-1}\mathbf{h}_1^i) \times (K^{-1}\mathbf{h}_2^i)$

- 迭代优化:

- 以上述 K, R_i, \mathbf{t}_i 的线性解 (代数误差最小化解) 为初值

- 对如下反投影误差最小化问题进行迭代求解:

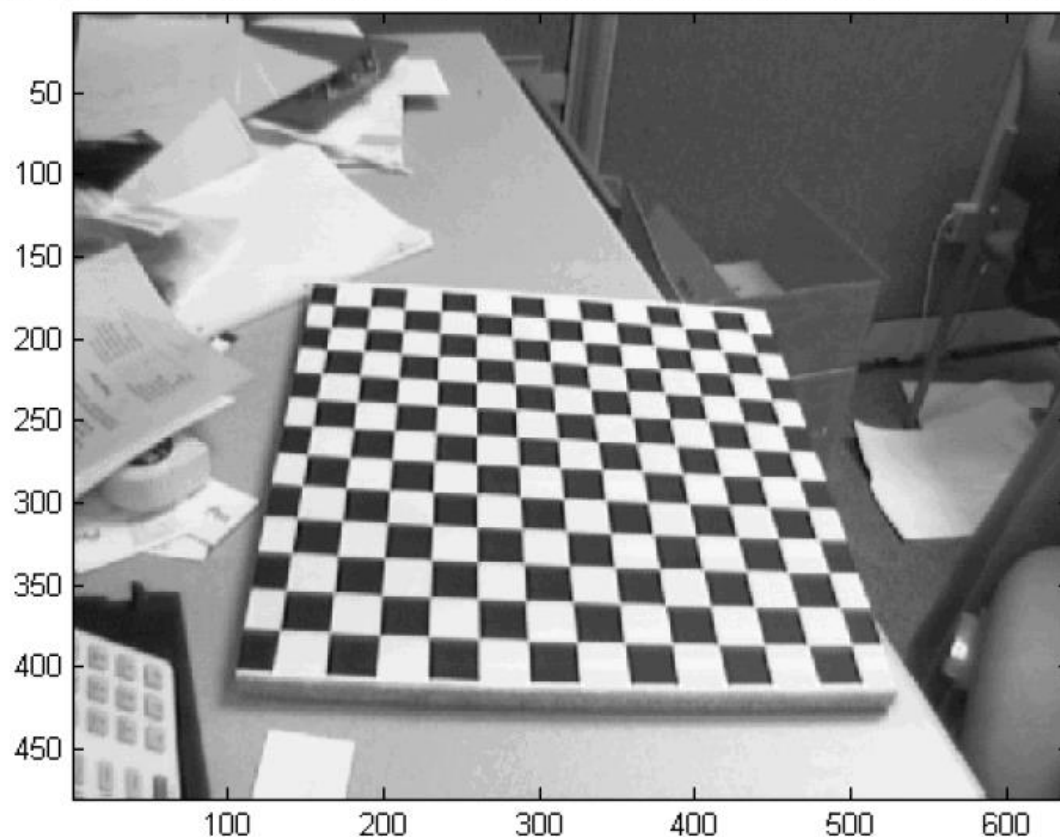
$$K^*, \{R_i^*, \mathbf{t}_i^*\} = \operatorname{argmin} \sum_{i,j} \|\mathbf{x}_{i,j} - K[R_i | \mathbf{t}_i] \mathbf{X}_{i,j}\|_2$$

- 即可获得几何误差最小化意义下的相机内外参数

相机标定 (Camera Calibration)

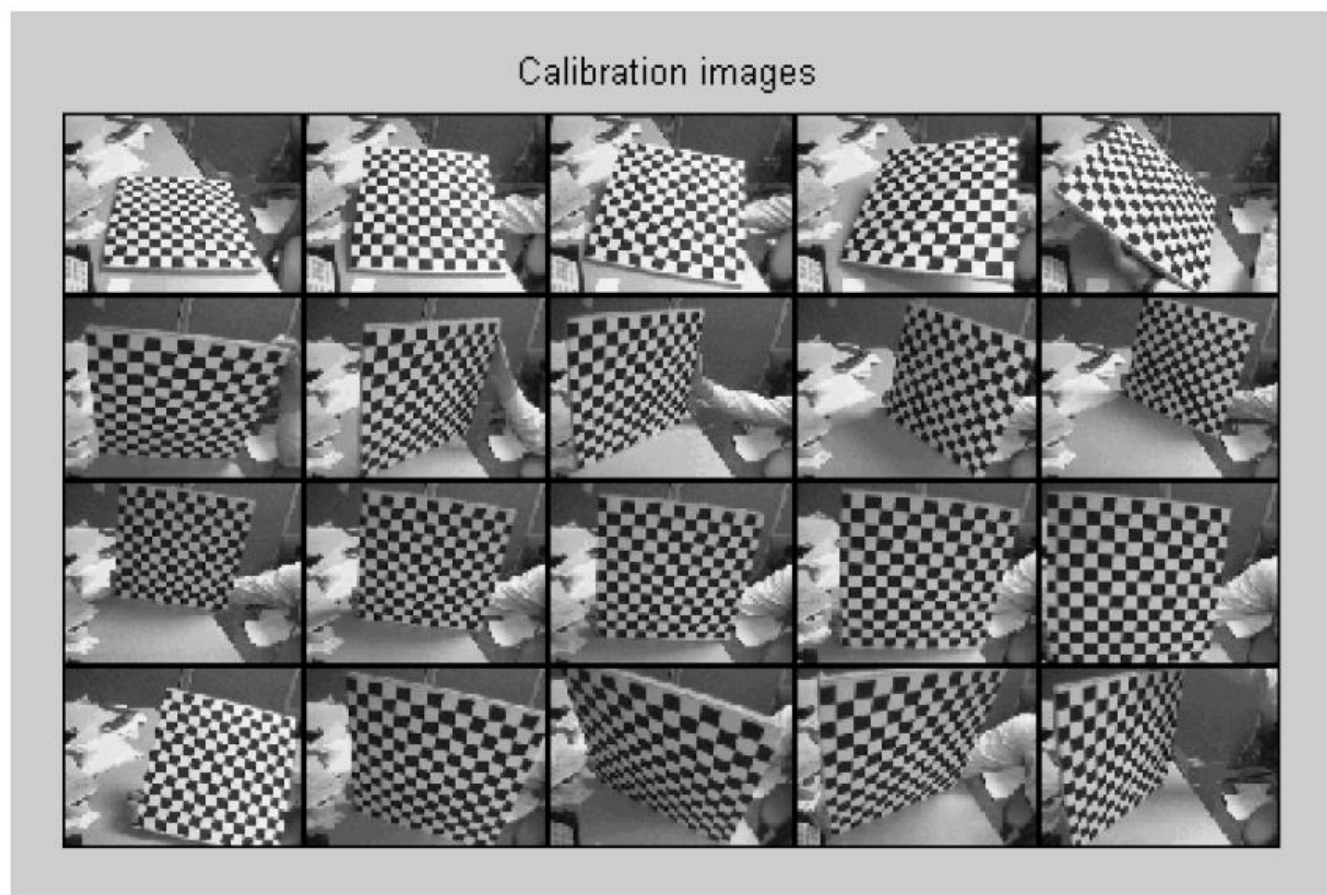
- 标定流程1: 打印一张模板并贴在一个平面上

Click on the four extreme corners of the rectangular pattern (first corner = origin)... Image 1



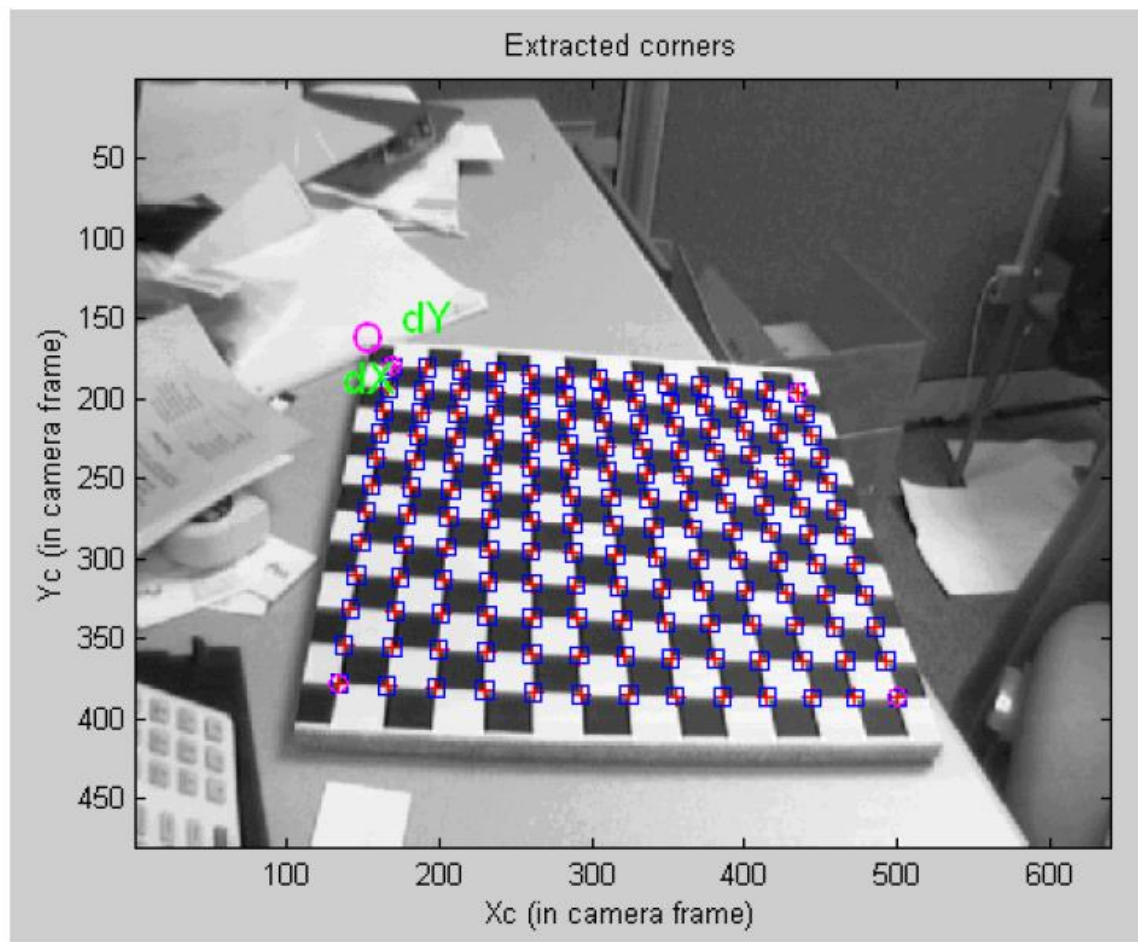
相机标定 (Camera Calibration)

- 标定流程2: 从不同角度拍摄若干张模板图像



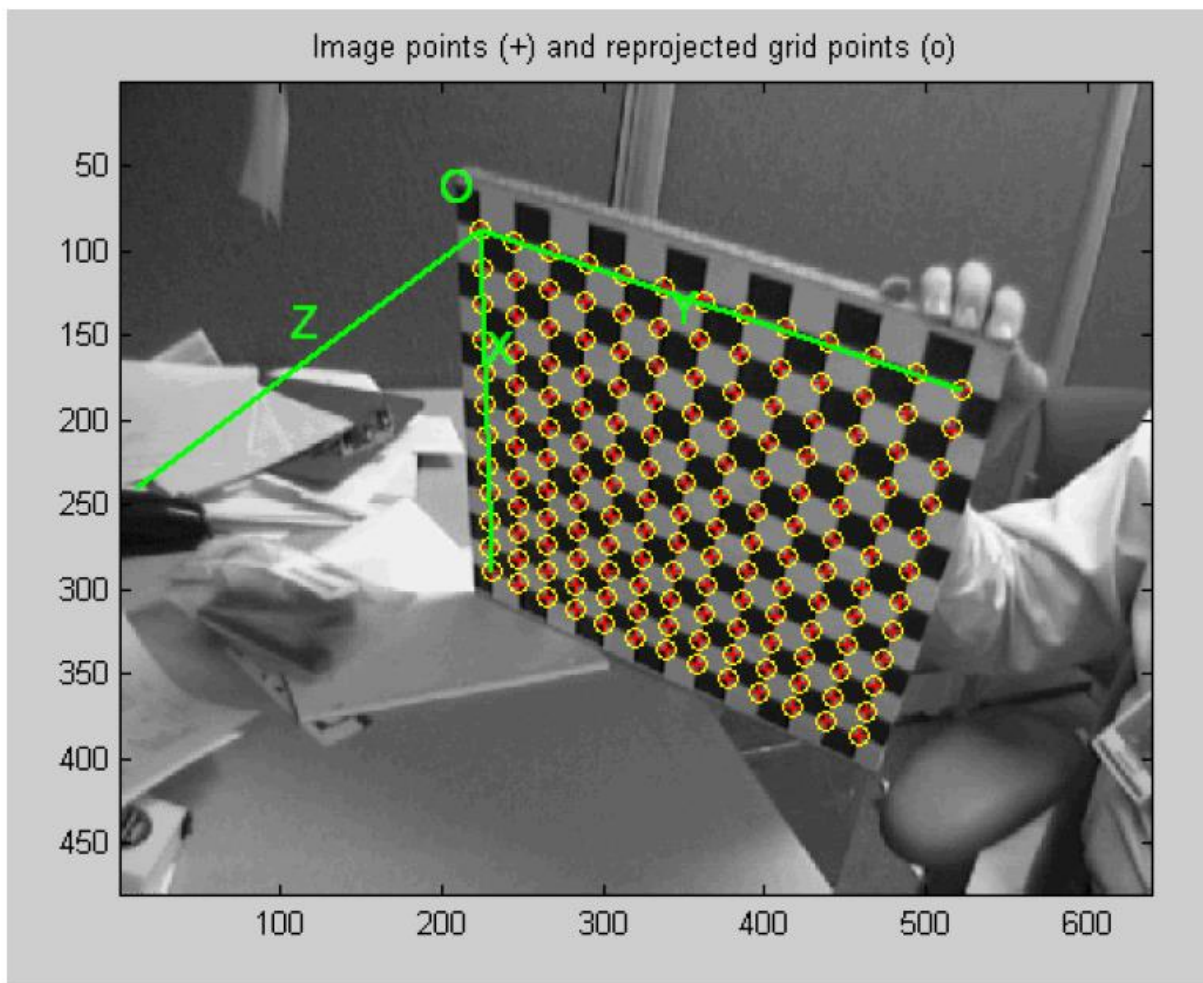
相机标定 (Camera Calibration)

- 标定流程3: 检测图像中的特征点



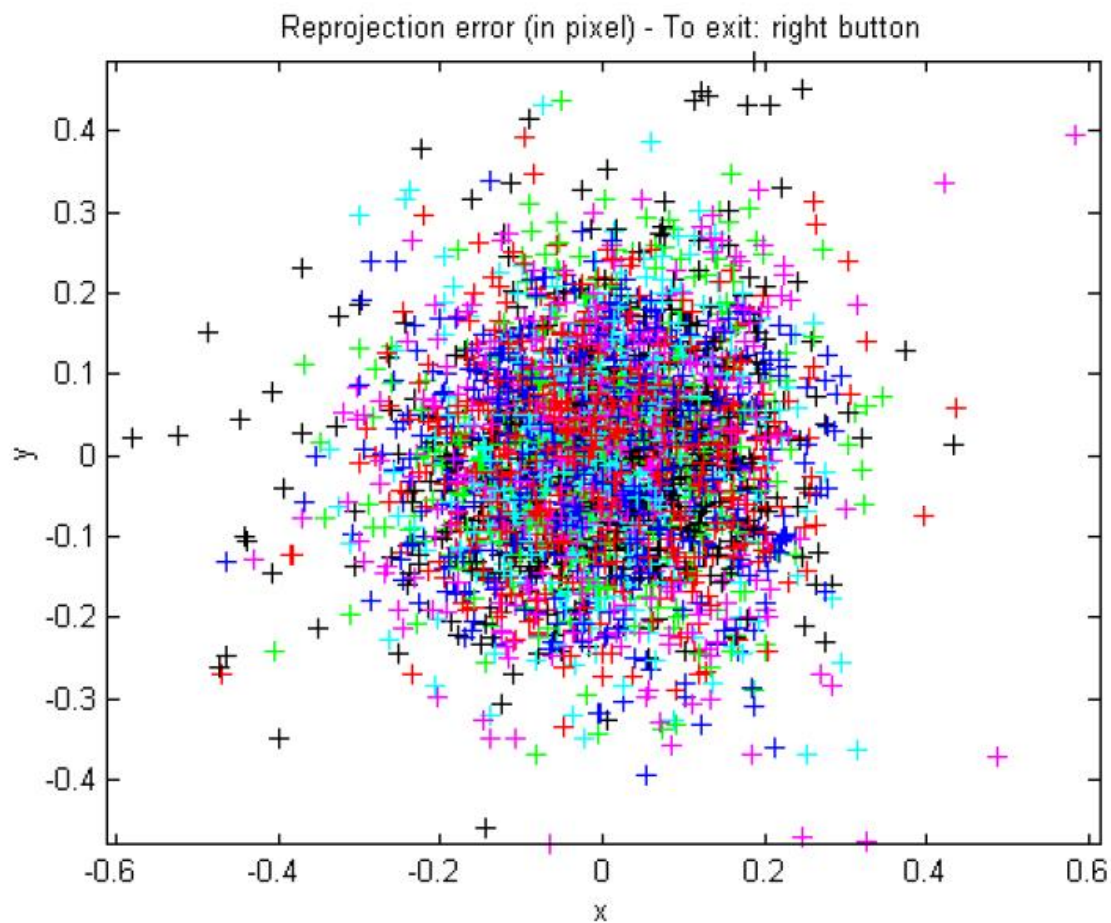
相机标定 (Camera Calibration)

- 标定流程4: 求解相机内外参数



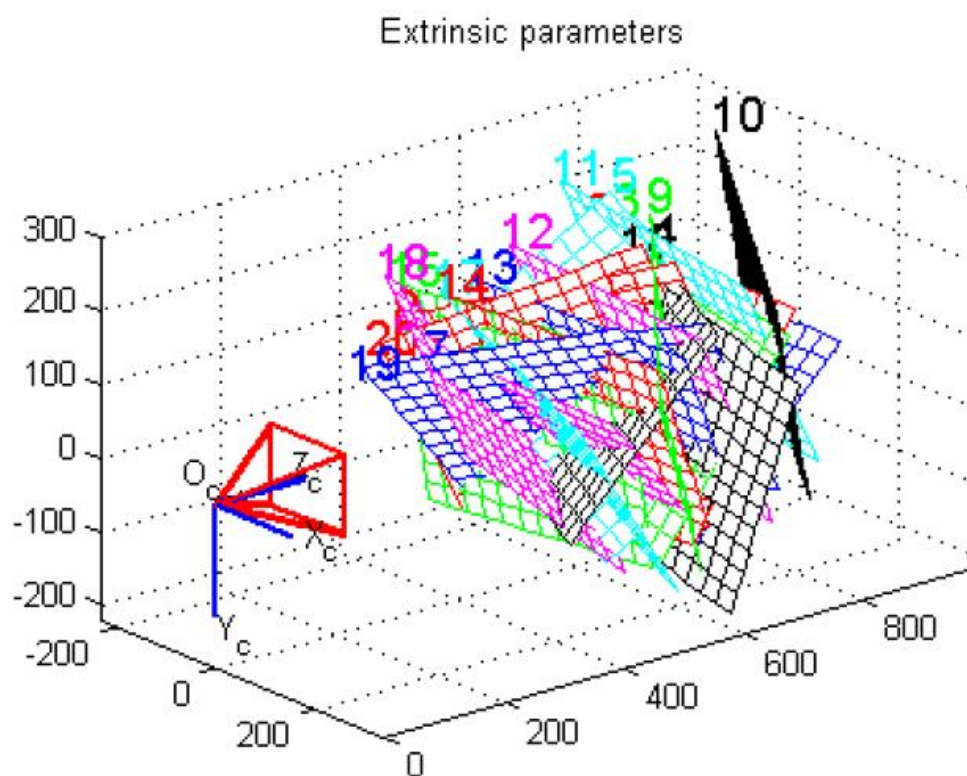
相机标定 (Camera Calibration)

- 标定流程5: 分析重投影误差



相机标定 (Camera Calibration)

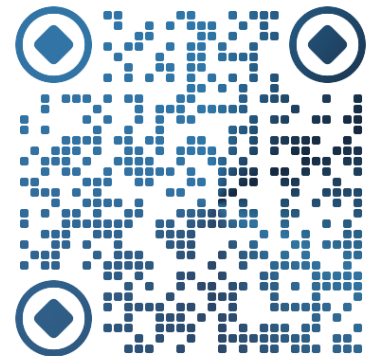
- 标定流程6: 输出标定结果



A Brief Introduction to 3D Vision

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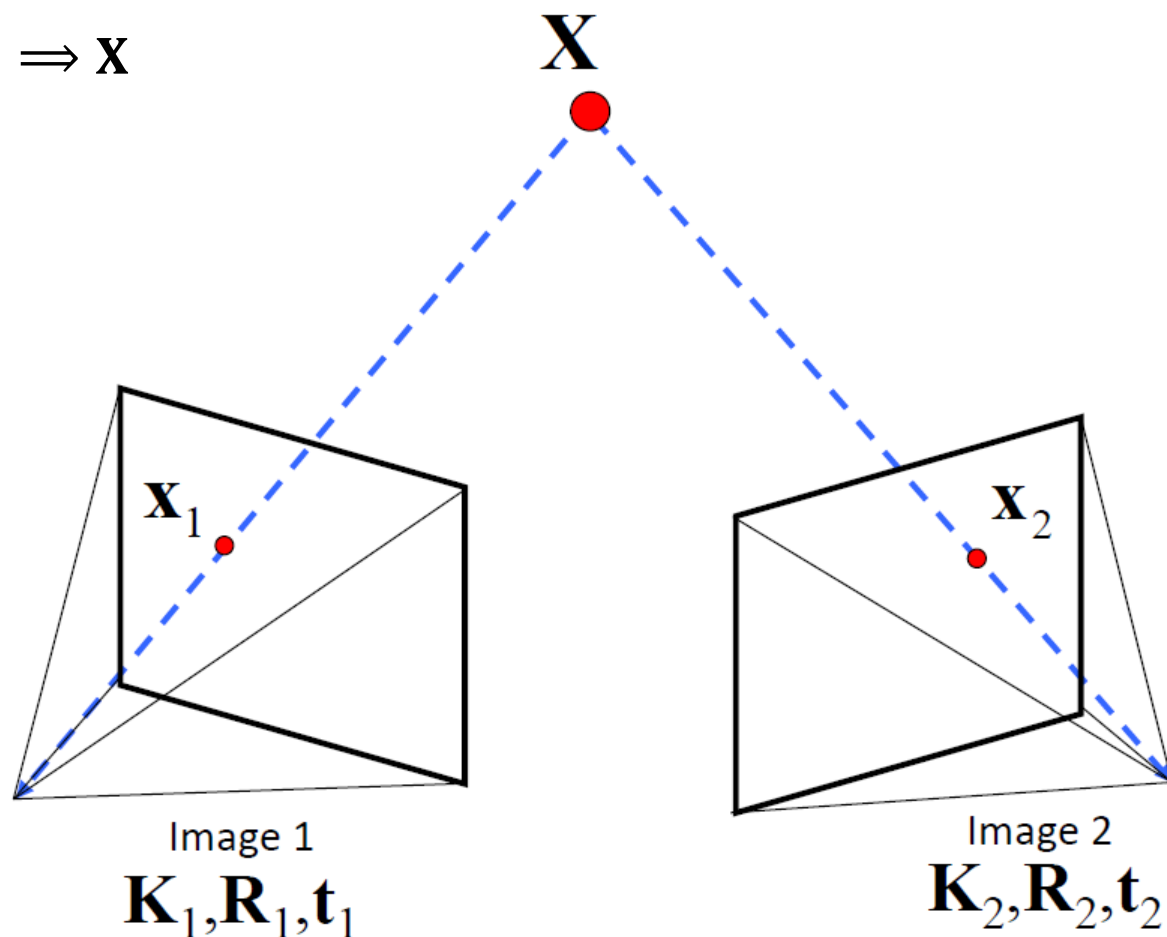
课程回顾

- 主要任务
 - 传感器, 数据, 重建, 理解
- 相机模型
 - 基本概念, 针孔模型, 主点偏置, 旋转平移, 畸变模型
- 相机标定
 - 基本概念
 - 基于三维靶标的标定方法
 - $\mathbf{x}_i \leftrightarrow \mathbf{X}_i \Rightarrow \mathbf{K}, \mathbf{R}, \mathbf{t}$
 - 基于二维靶标的标定方法
 - $\mathbf{x}_{i,j} \leftrightarrow \mathbf{X}_{i,j} \Rightarrow \mathbf{K}, \mathbf{R}_i, \mathbf{t}_i$

三角测量 (Triangulation)

- 基本概念

- 已知多幅图像中的二维对应点及各相机的内/外参数
- 求解二维图像点对应的三维空间点
- $\mathbf{x}_i, P_i \Rightarrow \mathbf{X}$



三角测量 (Triangulation)

- 具体流程

- 空间点 \mathbf{X} 在相机 P_i 上的投影记为 \mathbf{x}_i , 则:

- $$\lambda_i \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix} = \begin{bmatrix} p_{11}^i & p_{12}^i & p_{13}^i & p_{14}^i \\ p_{21}^i & p_{22}^i & p_{23}^i & p_{24}^i \\ p_{31}^i & p_{32}^i & p_{33}^i & p_{34}^i \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \Rightarrow$$

$$\begin{cases} \lambda_i x_i = p_{11}^i X + p_{12}^i Y + p_{13}^i Z + p_{14}^i \\ \lambda_i y_i = p_{21}^i X + p_{22}^i Y + p_{23}^i Z + p_{24}^i \\ \lambda_i = p_{31}^i X + p_{32}^i Y + p_{33}^i Z + p_{34}^i \end{cases} \Rightarrow$$

$$\begin{cases} (p_{31}^i X + p_{32}^i Y + p_{33}^i Z + p_{34}^i) x_i = p_{11}^i X + p_{12}^i Y + p_{13}^i Z + p_{14}^i \\ (p_{31}^i X + p_{32}^i Y + p_{33}^i Z + p_{34}^i) y_i = p_{21}^i X + p_{22}^i Y + p_{23}^i Z + p_{24}^i \end{cases}$$

三角测量 (Triangulation)

- 具体流程

- 上式写成矩阵相乘的形式:

- $$\begin{bmatrix} p_{31}^i x_i - p_{11}^i & p_{32}^i x_i - p_{12}^i & p_{33}^i x_i - p_{13}^i \\ p_{31}^i y_i - p_{21}^i & p_{32}^i y_i - p_{22}^i & p_{33}^i y_i - p_{23}^i \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} p_{14}^i - p_{34}^i x_i \\ p_{24}^i - p_{34}^i y_i \end{pmatrix}$$

- 将空间点 \mathbf{X} 在 $n(n \geq 2)$ 个相机下的投影约束关系堆叠在一起, 可得:

- $$\begin{bmatrix} p_{31}^1 x_1 - p_{11}^1 & p_{32}^1 x_1 - p_{12}^1 & p_{33}^1 x_1 - p_{13}^1 \\ p_{31}^1 y_1 - p_{21}^1 & p_{32}^1 y_1 - p_{22}^1 & p_{33}^1 y_1 - p_{23}^1 \\ \vdots & \vdots & \vdots \\ p_{31}^n x_n - p_{11}^n & p_{32}^n x_n - p_{12}^n & p_{33}^n x_n - p_{13}^n \\ p_{31}^n y_n - p_{21}^n & p_{32}^n y_n - p_{22}^n & p_{33}^n y_n - p_{23}^n \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} p_{14}^1 - p_{34}^1 x_i \\ p_{24}^1 - p_{34}^1 y_i \\ \vdots \\ p_{14}^n - p_{34}^n x_i \\ p_{24}^n - p_{34}^n y_i \end{pmatrix}$$

- 写成矩阵运算形式: $A(2n \times 3)\mathbf{X}(3 \times 1) = \mathbf{b}(2n \times 1)$

- 则 \mathbf{X} 的 (代数误差意义下的) 最小二乘解为:

- $\mathbf{X}_{\text{algebraic}} = \mathbf{A}^+ \mathbf{b} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{b}$

三角测量 (Triangulation)

- 具体流程

- 迭代优化:

- 以 $\mathbf{X}_{\text{algebraic}}$ 为初值

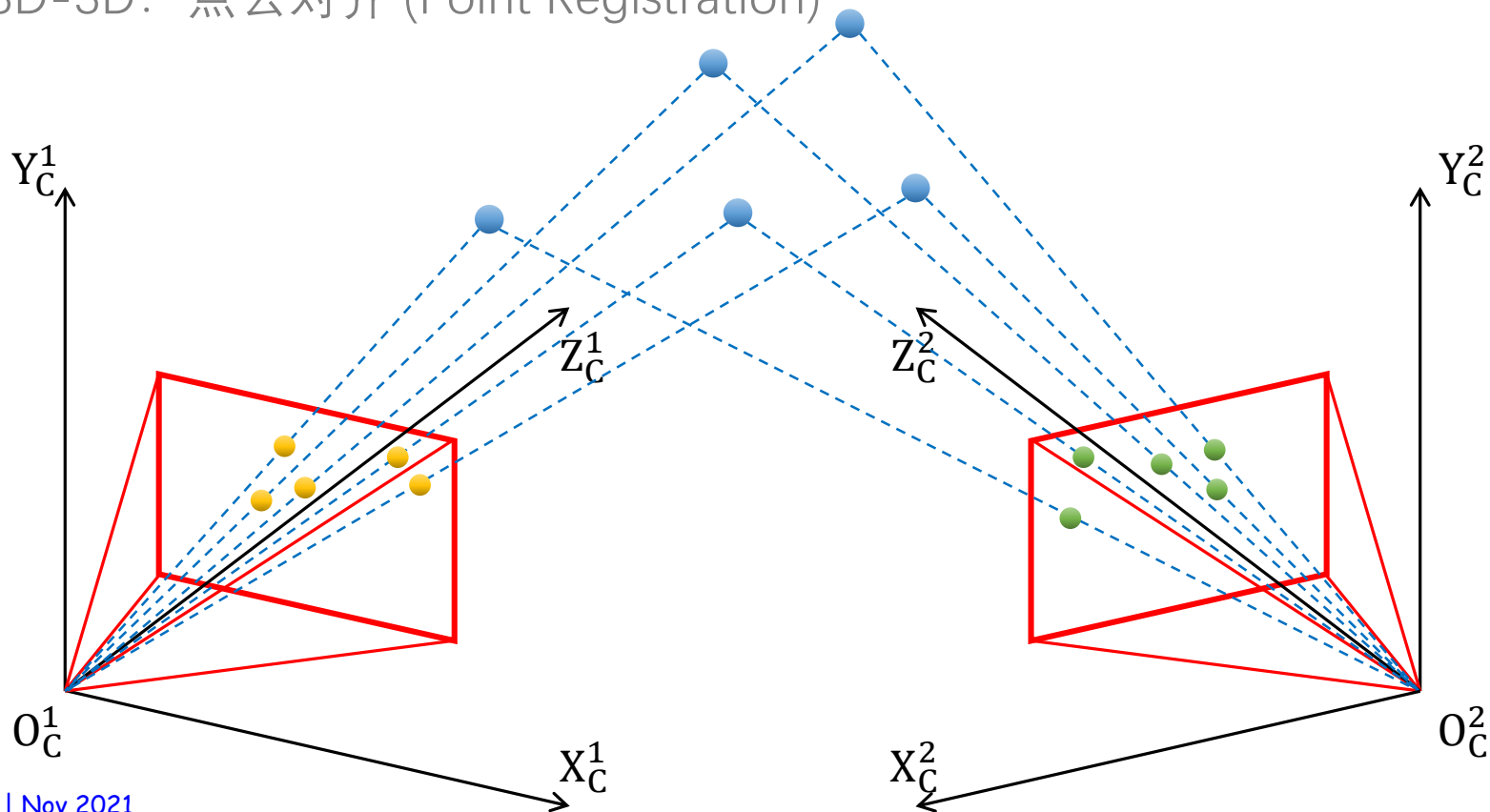
- 对如下反投影误差最小化问题进行迭代求解:

$$\mathbf{X}^* = \operatorname{argmin} \sum_{i=1}^n \|\mathbf{x}_i - P_i \mathbf{X}\|_2$$

- 即可获得几何误差最小化意义下的空间点三维坐标 $\mathbf{X}_{\text{geometric}}$

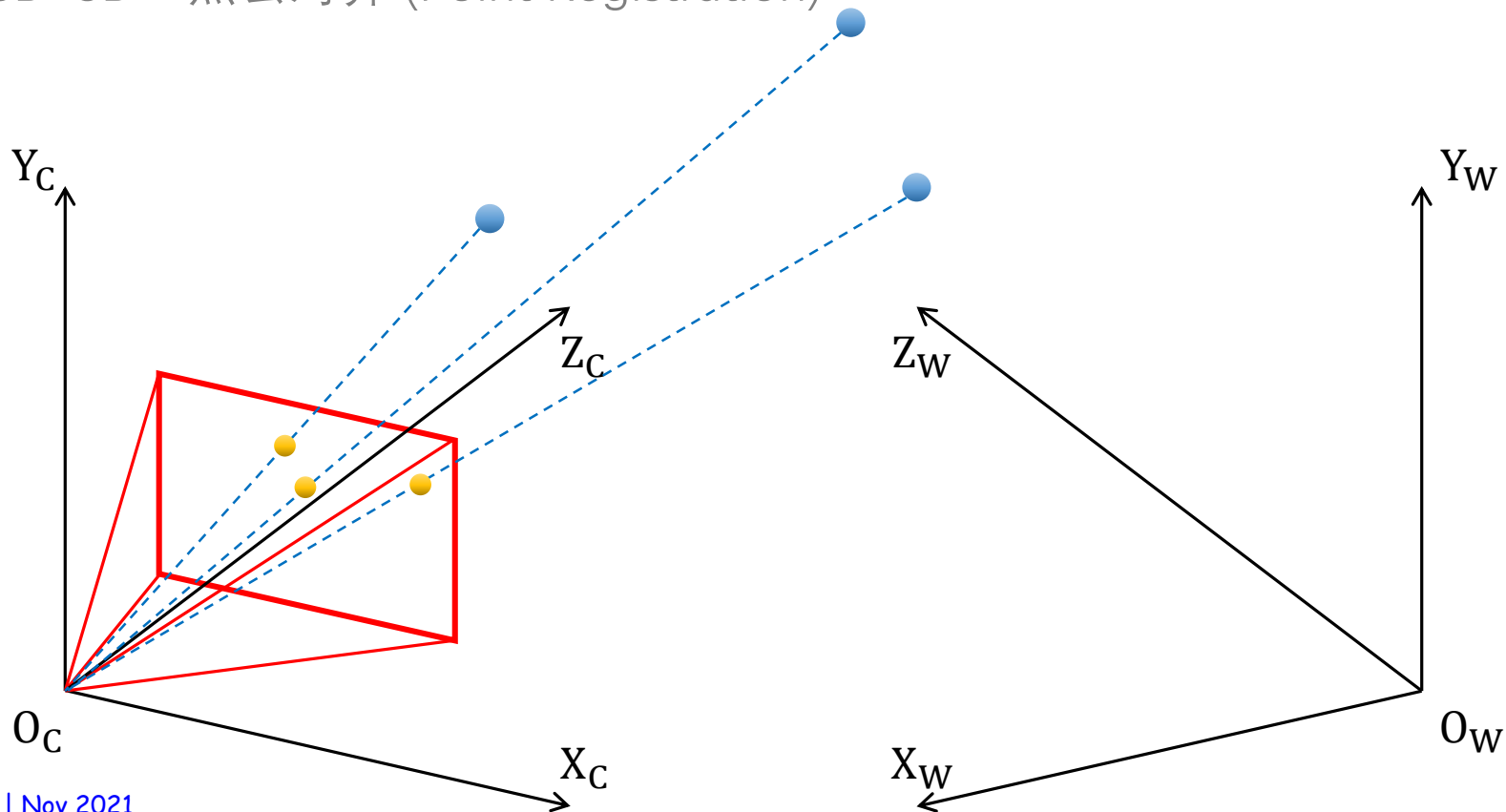
位姿估计

- 2D-2D: 相对定向 (**Relative Orientation**)
 - $\mathbf{x}_i^1 \leftrightarrow \mathbf{x}_i^2 \Rightarrow R_{12}, \mathbf{t}_{12} (O_C^1 - X_C^1 Y_C^1 Z_C^1 \rightarrow O_C^2 - X_C^2 Y_C^2 Z_C^2)$
- 2D-3D: 视觉定位 (Visual Localization)
- 3D-3D: 点云对齐 (Point Registration)



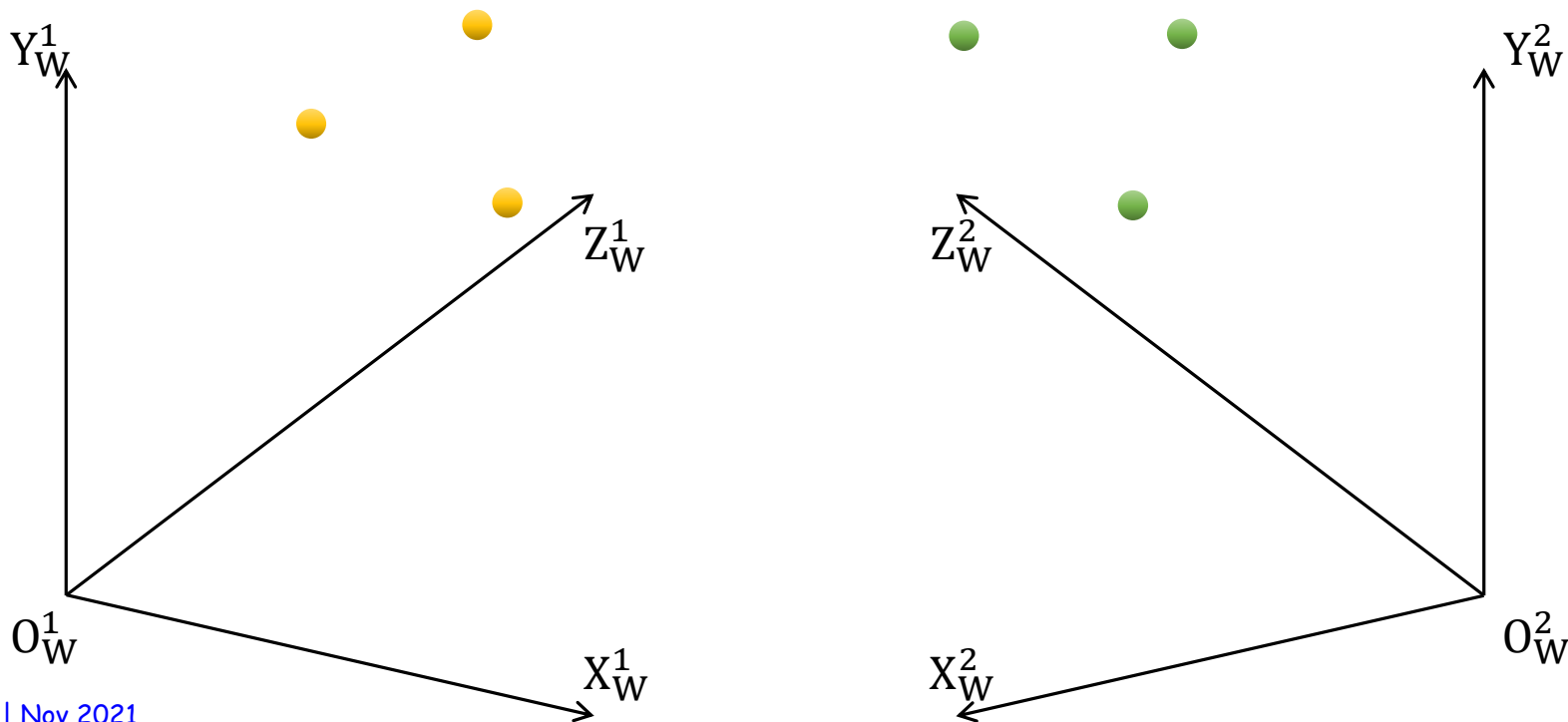
位姿估计

- 2D-2D: 相对定向 (Relative Orientation)
- **2D-3D: 视觉定位 (Visual Localization)**
 - $\mathbf{x}_i \leftrightarrow \mathbf{X}_i \Rightarrow R_{WC}, \mathbf{t}_{WC} (O_W - X_W Y_W Z_W \rightarrow O_C - X_C Y_C Z_C)$
- 3D-3D: 点云对齐 (Point Registration)



位姿估计

- 2D-2D: 相对定向 (Relative Orientation)
- 2D-3D: 视觉定位 (Visual Localization)
- **3D-3D: 点云对齐 (Point Registration)**
 - $\mathbf{X}_i^1 \leftrightarrow \mathbf{X}_i^2 \Rightarrow R_{12}, \mathbf{t}_{12} (O_W^1 - X_W^1 Y_W^1 Z_W^1 \rightarrow O_W^2 - X_W^2 Y_W^2 Z_W^2)$

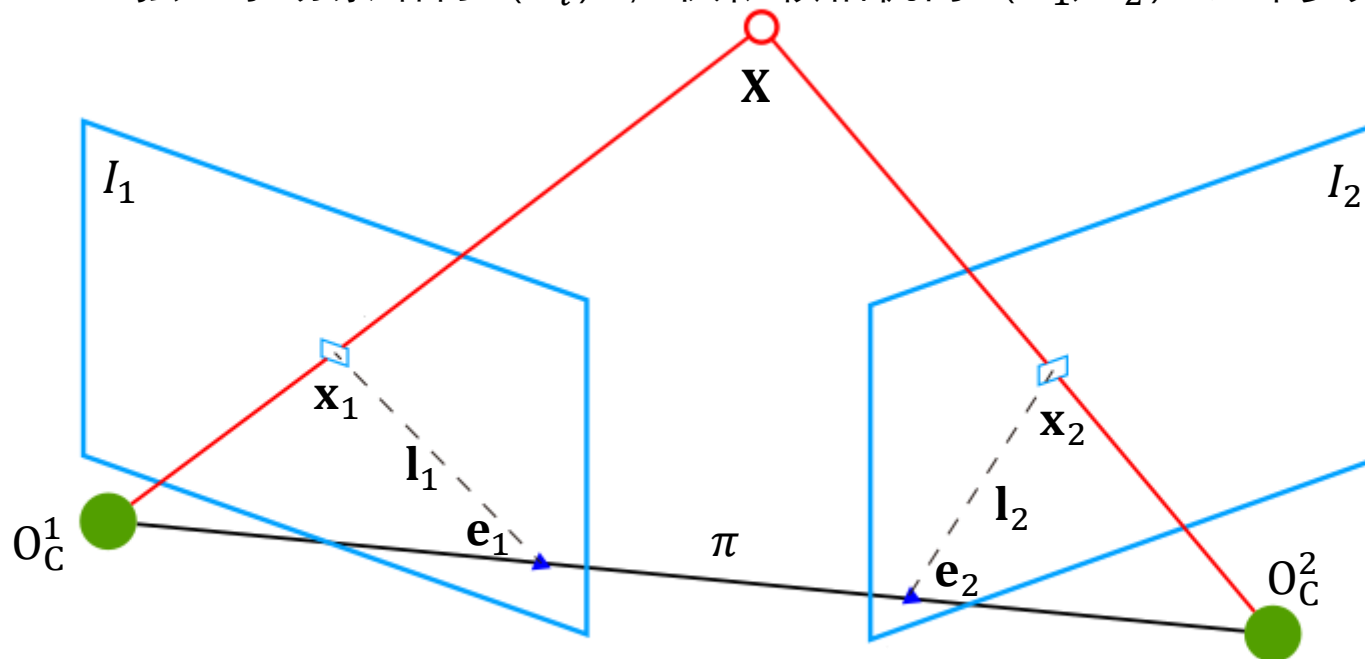


位姿估计

- 2D-2D: 相对定向

- 对极几何 (Epipolar Geometry)

- 用于刻画两视图匹配对 ($\mathbf{x}_i^1 \leftrightarrow \mathbf{x}_i^2$) 之间的内在几何关系
- 独立于场景结构 (\mathbf{X}_i), 仅依赖相机内 (K_1, K_2)、外参数 (R_{12}, \mathbf{t}_{12})



π : 对极平面 (Epipolar Plane); e_1, e_2 : 对极点 (Epipole); l_1, l_2 : 对极线 (Epipolar Line)

位姿估计

- 2D-2D: 对极几何 (Epipolar Geometry)

- 过程推导

- 假设世界坐标系建立在左相机上, 且两相机之间的变换关系记为 R, \mathbf{t}

- (1): $\lambda_1 \mathbf{x}_1 = K_1 [I | \mathbf{0}] \mathbf{X} = K_1 \tilde{\mathbf{X}}$

- (2): $\lambda_2 \mathbf{x}_2 = K_2 [R | \mathbf{t}] \mathbf{X} = K_2 R \tilde{\mathbf{X}} + K_2 \mathbf{t}$

- (3): $\tilde{\mathbf{X}} = \lambda_1 K_1^{-1} \mathbf{x}_1$ 式(1)变形

- (4): $\lambda_2 \mathbf{x}_2 = \lambda_1 K_2 R K_1^{-1} \mathbf{x}_1 + K_2 \mathbf{t}$ 式(3)带入式(2)

- (5): $\lambda_2 K_2^{-1} \mathbf{x}_2 = \lambda_1 R K_1^{-1} \mathbf{x}_1 + \mathbf{t}$ 式(4)两边左乘 K_2^{-1}

- (6): $\lambda_2 \mathbf{t} \times (K_2^{-1} \mathbf{x}_2) = \lambda_1 \mathbf{t} \times (R K_1^{-1} \mathbf{x}_1)$ \mathbf{t} 与式(5)两边做向量积

- (7): $\lambda_2 \mathbf{x}_2^T K_2^{-T} \mathbf{t} \times (K_2^{-1} \mathbf{x}_2) = \lambda_1 \mathbf{x}_2^T K_2^{-T} \mathbf{t} \times (R K_1^{-1} \mathbf{x}_1)$ 式(6)两边左乘 $\mathbf{x}_2^T K_2^{-T}$

- (8): $\mathbf{x}_2^T K_2^{-T} \mathbf{t} \times (K_2^{-1} \mathbf{x}_2) = 0$ $\mathbf{t} \times (K_2^{-1} \mathbf{x}_2)$ 结果与 $K_2^{-1} \mathbf{x}_2$ 正交

- (9): $\mathbf{x}_2^T K_2^{-T} \mathbf{t} \times (R K_1^{-1} \mathbf{x}_1) = 0$ 式(8)带入式(7)

- (10): $\mathbf{x}_2^T \underbrace{K_2^{-T} [\mathbf{t}]_{\times} R K_1^{-1}}_F \mathbf{x}_1 = 0$ $[\mathbf{t}]_{\times} = \begin{bmatrix} 0 & -t_3 & t_2 \\ t_3 & 0 & -t_1 \\ -t_2 & t_1 & 0 \end{bmatrix}, \mathbf{t} \times \mathbf{x} = [\mathbf{t}]_{\times} \mathbf{x}$

位姿估计

- 2D-2D: 对极几何 (Epipolar Geometry)

- 基本矩阵 (Fundamental Matrix)

- F is a rank 2 homogeneous matrix with 7 degrees of freedom.
- **Point correspondence:** If \mathbf{x} and \mathbf{x}' are corresponding image points, then $\mathbf{x}'^T F \mathbf{x} = 0$.
- **Epipolar lines:**
 - ◇ $\mathbf{l}' = F \mathbf{x}$ is the epipolar line corresponding to \mathbf{x} .
 - ◇ $\mathbf{l} = F^T \mathbf{x}'$ is the epipolar line corresponding to \mathbf{x}' .
- **Epipoles:**
 - ◇ $F \mathbf{e} = \mathbf{0}$.
 - ◇ $F^T \mathbf{e}' = \mathbf{0}$.
- **Computation from camera matrices P, P' :**
 - ◇ General cameras, $F = [\mathbf{e}']_{\times} P' P^+$, where P^+ is the pseudo-inverse of P , and $\mathbf{e}' = P' \mathbf{C}$, with $P \mathbf{C} = \mathbf{0}$.
 - ◇ Canonical cameras, $P = [I \mid \mathbf{0}]$, $P' = [M \mid \mathbf{m}]$, $F = [\mathbf{e}']_{\times} M = M^{-T} [\mathbf{e}]_{\times}$, where $\mathbf{e}' = \mathbf{m}$ and $\mathbf{e} = M^{-1} \mathbf{m}$.
 - ◇ Cameras not at infinity $P = K[I \mid \mathbf{0}]$, $P' = K'[R \mid \mathbf{t}]$, $F = K'^{-T} [\mathbf{t}]_{\times} R K^{-1} = [K' \mathbf{t}]_{\times} K' R K^{-1} = K'^{-T} R K^T [K R^T \mathbf{t}]_{\times}$.

位姿估计

- 2D-2D: 对极几何 (Epipolar Geometry)
 - 本质矩阵 (Essential Matrix)
 - 当相机内参数已知时, 对其成像点进行归一化:
 - $\mathbf{x}_1 \leftrightarrow \mathbf{x}_2 \Rightarrow \hat{\mathbf{x}}_1 = \mathbf{K}_1^{-1}\mathbf{x}_1 \leftrightarrow \hat{\mathbf{x}}_2 = \mathbf{K}_2^{-1}\mathbf{x}_2$, 则:
 - $\mathbf{x}_2^T \mathbf{K}_2^{-T} [\mathbf{t}]_{\times} \mathbf{R} \mathbf{K}_1^{-1} \mathbf{x}_1 = \hat{\mathbf{x}}_2^T \underbrace{[\mathbf{t}]_{\times} \mathbf{R}}_{\mathbf{E}} \hat{\mathbf{x}}_1$
 - 本质矩阵的性质
 - 本质矩阵的自由度为5
 - 本质矩阵的奇异值中有两个相等而第三个是0

位姿估计

- 2D-2D: 对极几何 (Epipolar Geometry)

- 由本质矩阵恢复摄像机矩阵

- $E, K_1, K_2 \Rightarrow P_1, P_2$

- 对E进行SVD: $E = U \text{diag}(\sigma \ \sigma \ 0) V^T$, 则:

- $P_1 = K_1 [I | \mathbf{0}]$, P_2 有下列四种可能的选择:

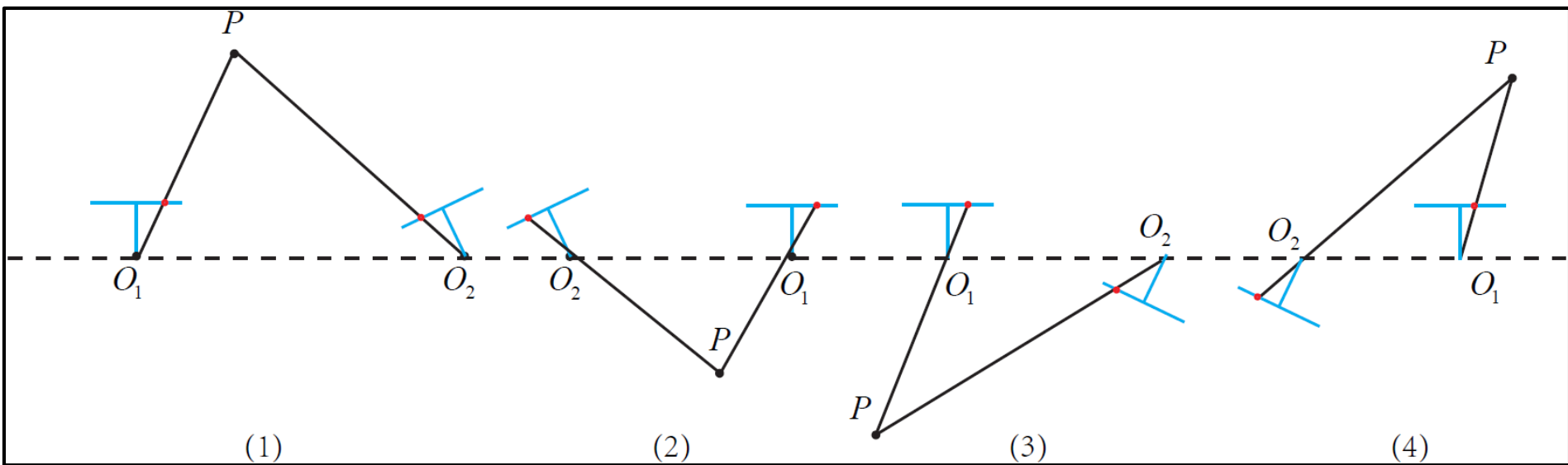
- $$P_2 = \begin{cases} K_2 [UWV^T | +\mathbf{u}_3] \\ K_2 [UWV^T | -\mathbf{u}_3] \\ K_2 [UW^T V^T | +\mathbf{u}_3] \\ K_2 [UW^T V^T | -\mathbf{u}_3] \end{cases}$$

- 其中,
$$W = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

位姿估计

- 2D-2D: 对极几何 (Epipolar Geometry)

- 四种解的几何解释:
$$P_2 = \begin{cases} K_2[UWV^T|+\mathbf{u}_3] \\ K_2[UWV^T|-\mathbf{u}_3] \\ K_2[UW^TV^T|+\mathbf{u}_3] \\ K_2[UW^TV^T|-\mathbf{u}_3] \end{cases}$$



位姿估计

- 2D-2D: 对极几何 (Epipolar Geometry)

- 基本矩阵的估计: $\mathbf{x}_i^1 \leftrightarrow \mathbf{x}_i^2 \Rightarrow \mathbf{F}$

- 一对2D-2D对应点可提供如下约束关系:

- $$\begin{pmatrix} x_i^2 \\ y_i^2 \\ 1 \end{pmatrix}^T \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{pmatrix} x_i^1 \\ y_i^1 \\ 1 \end{pmatrix} = 0 \Rightarrow$$

- $$x_i^2 x_i^1 f_{11} + x_i^2 y_i^1 f_{12} + x_i^2 f_{13} + y_i^2 x_i^1 f_{21} + y_i^2 y_i^1 f_{22} + y_i^2 f_{23} + x_i^1 f_{31} + y_i^1 f_{32} + f_{33} = 0$$

- 将上式写成矩阵相乘的形式:

- $$[x_i^2 x_i^1 \quad x_i^2 y_i^1 \quad x_i^2 \quad y_i^2 x_i^1 \quad y_i^2 y_i^1 \quad y_i^2 \quad x_i^1 \quad y_i^1 \quad 1] \begin{pmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{pmatrix} = 0$$

位姿估计

- 2D-2D: 对极几何 (Epipolar Geometry)

- 基本矩阵的估计: $\mathbf{x}_i^1 \leftrightarrow \mathbf{x}_i^2 \Rightarrow \mathbf{F}$

- 将 $n (n \geq 8)$ 对 2D-2D 对应点提供的约束关系堆叠在一起, 可得:

$$\bullet \begin{bmatrix} x_1^2 x_1^1 & x_1^2 y_1^1 & x_1^2 & y_1^2 x_1^1 & y_1^2 y_1^1 & y_1^2 & x_1^1 & y_1^1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_n^2 x_n^1 & x_n^2 y_n^1 & x_n^2 & y_n^2 x_n^1 & y_n^2 y_n^1 & y_n^2 & x_n^1 & y_n^1 & 1 \end{bmatrix} \begin{pmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{pmatrix} = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}$$

- 写成矩阵运算形式: $\mathbf{A} (n \times 9) \mathbf{f} (9 \times 1) = \mathbf{0} (n \times 1)$

- 对矩阵 \mathbf{A} 进行 SVD 获取 \mathbf{f} (代数误差意义下) 的最小二乘解 $\mathbf{f}_{\text{algebraic}}$

位姿估计

- 2D-2D: 对极几何 (Epipolar Geometry)

- 基本矩阵的估计: $\mathbf{x}_i^1 \leftrightarrow \mathbf{x}_i^2 \Rightarrow \mathbf{F}$

- 以 $\mathbf{f}_{\text{algebraic}}$ 为初值

- 对如下反投影误差最小化问题进行迭代求解:

$$\mathbf{F}^* = \operatorname{argmin} \sum_{i=1}^n [d^2(\mathbf{x}_i^2, \mathbf{F}\mathbf{x}_i^1) + d^2(\mathbf{x}_i^1, \mathbf{F}^T\mathbf{x}_i^2)]$$

- 其中, $d(\mathbf{x}, \mathbf{l})$ 表述点 \mathbf{x} 到线 \mathbf{l} 的欧氏距离

- 对上式优化即可获得几何误差最小化意义下的基本矩阵估计 $\mathbf{F}_{\text{geometric}}$

位姿估计

- 2D-2D: 对极几何 (Epipolar Geometry)

- 基本矩阵奇异性约束: $\text{rank}(F) = 2$ 即 $\sigma_3 = 0$

- $$\begin{cases} \min_{\hat{F}} \|F - \hat{F}\|_F \\ \text{s. t. rank}(\hat{F}) = 2 \end{cases}$$

- 其中, $\|A\|_F = \sqrt{\text{tr}(A^T A)} = \sqrt{\sum_{i=1}^m \sum_{j=1}^n a_{ij}^2}$

- 对F进行SVD: $F = U \text{diag}(\sigma_1 \ \sigma_2 \ \sigma_3) V^T$, 则:

- $\hat{F} = U \text{diag}(\sigma_1 \ \sigma_2 \ 0) V^T$

- 本质矩阵奇异性约束: $\sigma_1 = \sigma_2, \sigma_3 = 0$

- $$\begin{cases} \min_{\hat{E}} \|E - \hat{E}\|_F \\ \text{s. t. } \sigma_1 = \sigma_2, \sigma_3 = 0 \end{cases}$$

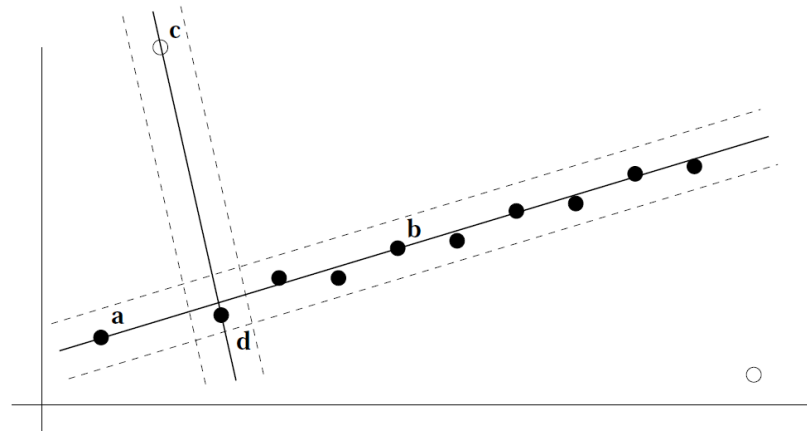
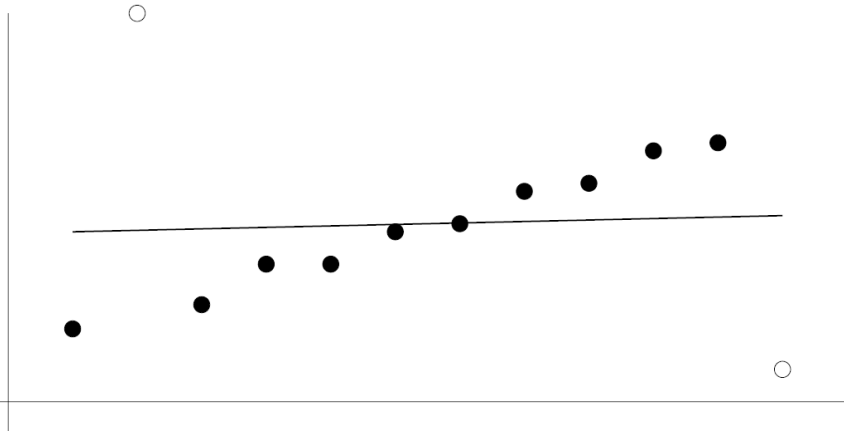
- $E = K_2^T F K_1$

- 对E进行SVD: $E = U \text{diag}(\sigma_1 \ \sigma_2 \ \sigma_3) V^T$, 则:

- $\hat{E} = U \text{diag}\left(\frac{\sigma_1 + \sigma_2}{2} \ \frac{\sigma_1 + \sigma_2}{2} \ 0\right) V^T$

位姿估计

- 2D-2D: 对极几何 (Epipolar Geometry)
 - 估计基本矩阵 vs. 错误匹配过滤
 - Chicken-and-egg Problem: RANdom SAmple Consensus, RANSAC



M. A. Fischler, R. C. Bolles. Random Sample Consensus: A Paradigm for Model Fitting with Applications to Image Analysis and Automated Cartography. *Communications of the ACM*, 1981, 24(6): 381-395. **27k+ Citations**



位姿估计

- 2D-2D: 对极几何 (Epipolar Geometry)
 - 主要流程:
 - 1): 从测量数据中随机抽取 s 组
 - s 为用于模型估计的最小集合大小
 - ① $P: s = 6$; ② $H: s = 4$; ③ $X: s = 2$; ④ $F: s = 7$; ⑤ $E: s = 5$
 - 2): 用随机抽取的数据进行模型估计
 - 3): 根据给定的度量函数与距离阈值获取当前模型的一致集合
 - ① P, H, X : 点到点距离; ② F, E : 点到线距离
 - 4): 重复1-3步 N 次以获取最大支撑集合
 - 5): 利用最大支撑集合进行模型估计

位姿估计

- 2D-2D: 对极几何 (Epipolar Geometry)

- RANSAC

- 基本思想:

- The RANSAC procedure is opposite to that of conventional smoothing techniques: Rather than using as much of the data as possible to obtain an initial solution and then attempting to eliminate the invalid data points, RANSAC uses as small an initial data set as feasible and enlarges this set with consistent data when possible.

- s 的重要性:

- $$N = \log(1 - p) / \log(1 - (1 - \epsilon)^s)$$

Sample size s	Proportion of outliers ϵ						
	5%	10%	20%	25%	30%	40%	50%
2	2	3	5	6	7	11	17
3	3	4	7	9	11	19	35
4	3	5	9	13	17	34	72
5	4	6	12	17	26	57	146
6	4	7	16	24	37	97	293
7	4	8	20	33	54	163	588
8	5	9	26	44	78	272	1177

位姿估计

- 2D-2D: 对极几何 (Epipolar Geometry)
 - RANSAC
 - N 的自适应决定:
 - $N = \log(1 - p) / \log(1 - (1 - \epsilon)^S)$

- $N = \infty$, sample_count = 0.
- While $N > \text{sample_count}$ Repeat
 - Choose a sample and count the number of inliers.
 - Set $\epsilon = 1 - (\text{number of inliers}) / (\text{total number of points})$
 - Set N from ϵ and (4.18) with $p = 0.99$.
 - Increment the sample_count by 1.
- Terminate.



位姿估计

- 2D-2D: 对极几何 (Epipolar Geometry)
 - 最小配置解
 - 基本矩阵: 7点法
 - R. I. Hartley. **Projective Reconstruction and Invariants from Multiple Images**. *IEEE Transactions on Pattern Analysis and Machine Intelligence (T-PAMI)*, 1994, 16(10): 1036-1041. **300+ Citations**
 - 本质矩阵: 5点法
 - D. Nister. **An Efficient Solution to the Five-Point Relative Pose Problem**. *IEEE Transactions on Pattern Analysis and Machine Intelligence (T-PAMI)*, 26(6): 2004, 756-770. **2k+ Citations**



位姿估计

- 2D-3D: 视觉定位

- 透视n点 (Perspective-n-Points, PnP)

- V. Lepetit, F. Moreno-Noguer, P. Fua. **EPnP: An Accurate $O(n)$ Solution to the PnP Problem.** *International Journal of Computer Vision (IJCV)*, 2009, 81(2): 155-166. **2k+ Citations**
 - X. Gao, X Hou, J Tang, et al. **Complete Solution Classification for the Perspective-Three-Point Problem.** *IEEE Transactions on Pattern Analysis and Machine Intelligence (T-PAMI)*, 2003, 25(8): 930-943. **800+ Citations**
 - J. A. Hesch, S. I. Roumeliotis. **A Direct Least-Squares (DLS) Method for PnP.** *In International Conference on Computer Vision (ICCV)*, 2011: 383-390. **300+ Citations**

- 3D-3D: 点云对齐

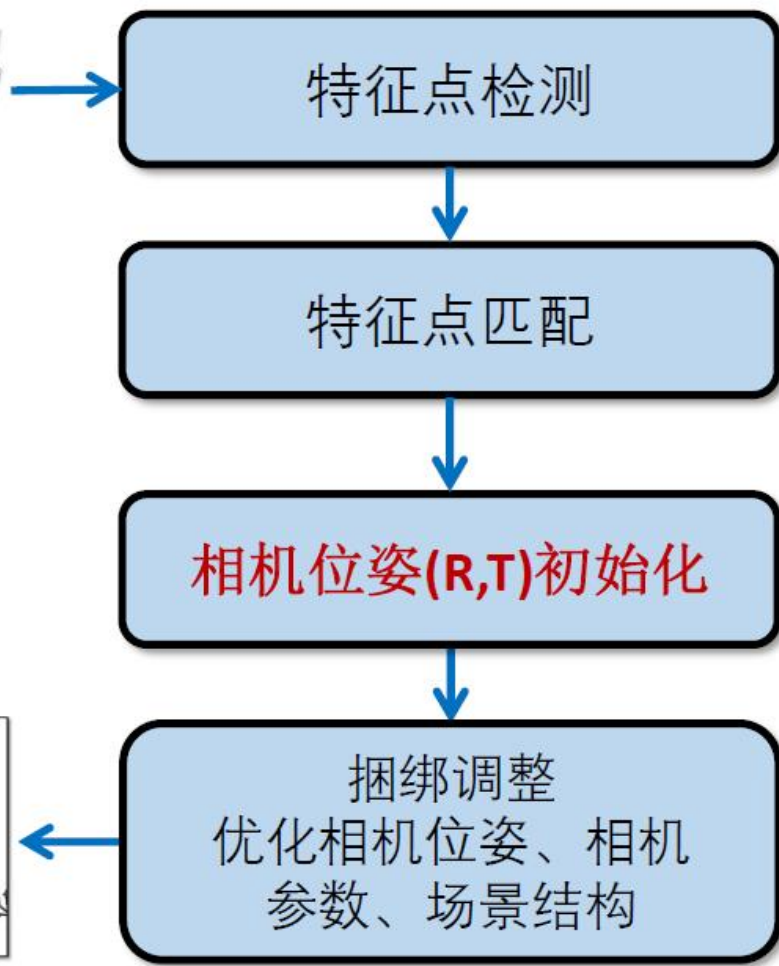
- 欧氏变换估计

- K. S. Arun, T. S. Huang, S. D. Blostein. **Least-Squares Fitting of Two 3-D Point Sets.** *IEEE Transactions on Pattern Analysis and Machine Intelligence (T-PAMI)*, 1987, 9(5): 698-700. **4k+ Citations**

- 相似变换估计

- S. Umeyama. **Least-Squares Estimation of Transformation Parameters Between Two Point Patterns.** *IEEE Transactions on Pattern Analysis and Machine Intelligence (T-PAMI)*, 1991, 13(4): 376-380. **2k+ Citations**

稀疏重建(Structure from Motion)

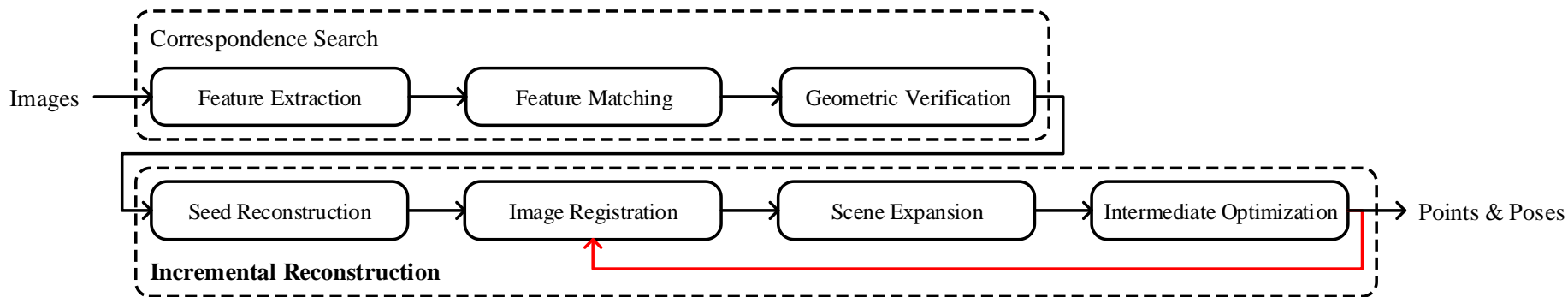


- 1. 增量式
- 2. 全局式
- 3. 混合式

从运动恢复结构 (Structure from Motion, SfM)

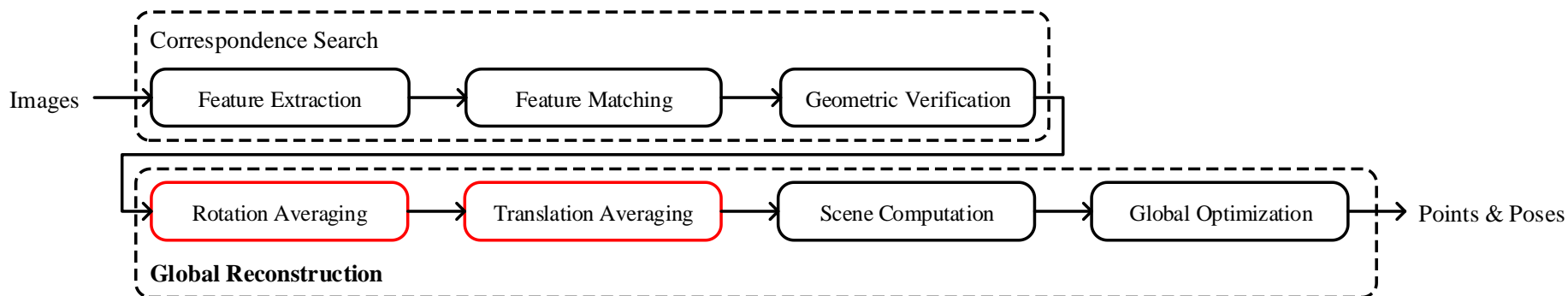


• 增量式SfM



- N. Snavely, S. M. Seitz, R. Szeliski. [Modeling the World from Internet Photo Collections](#). *International Journal of Computer Vision (IJCV)*, 2009, 80(2): 189-210. **2k+ Citations**
- J. L. Schönberger, J. M. Frahm. [Structure from Motion Revisited](#). In *IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*, 2016: 4104-4113. **1k+ Citations**

• 全局式SfM

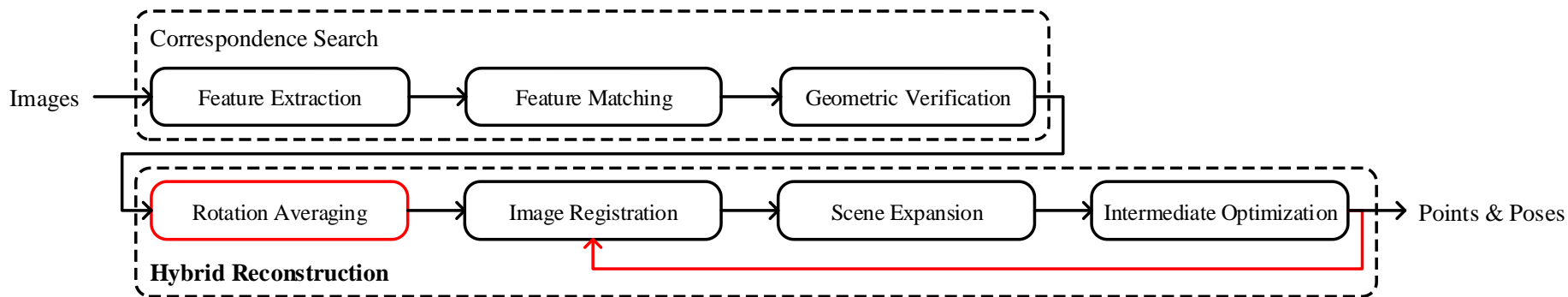


- A. Chatterjee, V. M. Govindu N. [Efficient and Robust Large-Scale Rotation Averaging](#). In *IEEE International Conference on Computer Vision (ICCV)*, 2013: 521-528. **200+ Citations**
- Z. Cui, P. Tan. [Global Structure-from-Motion by Similarity Averaging](#). In *IEEE International Conference on Computer Vision (ICCV)*, 2015: 864-872. **100+ Citations**

从运动恢复结构 (Structure from Motion, SfM)



- 混合增量式SfM



- H. Cui, X. Gao, S. Shen, et al. **HSfM: Hybrid Structure-from-Motion**. In *IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*, 2017: 521-528. **70+ Citations**

Photo Tourism — 两视图SfM

- 5点法

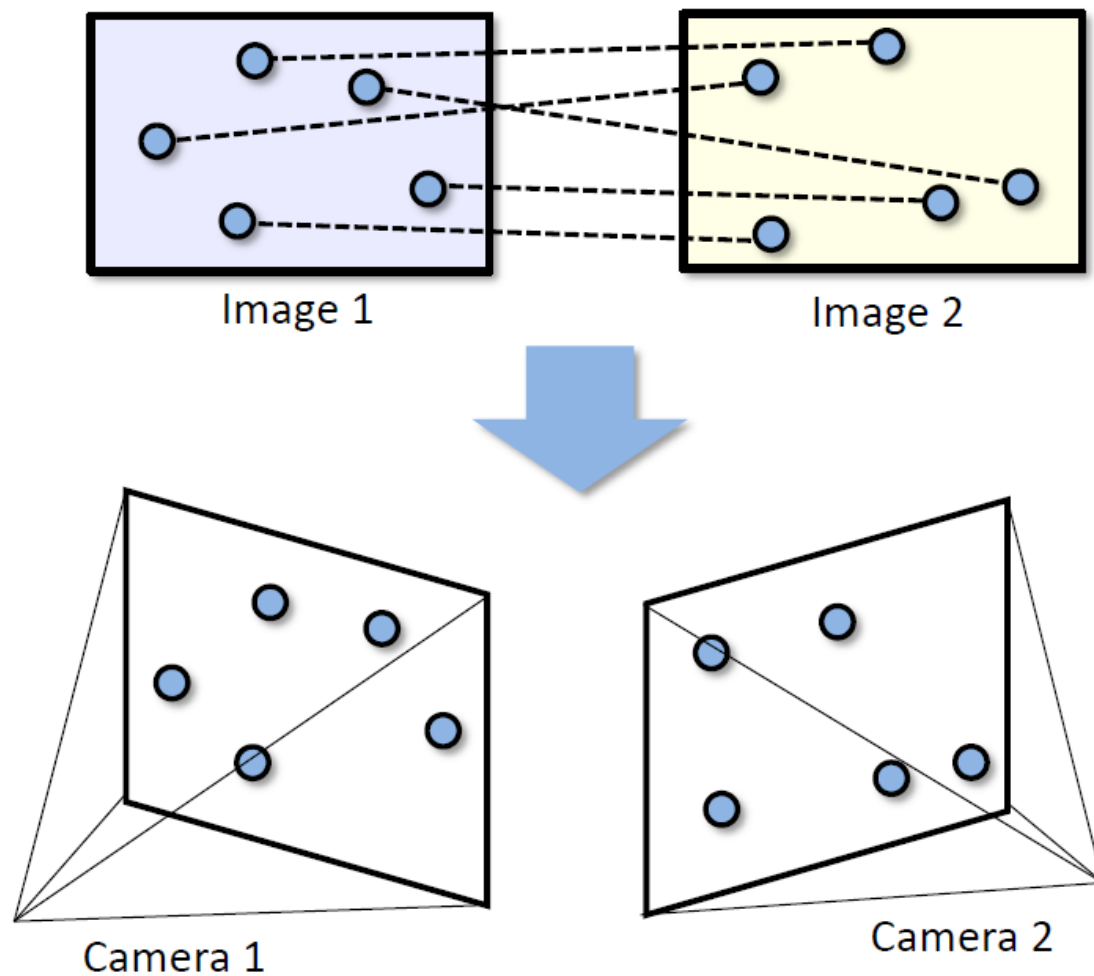


Photo Tourism — 两视图SfM

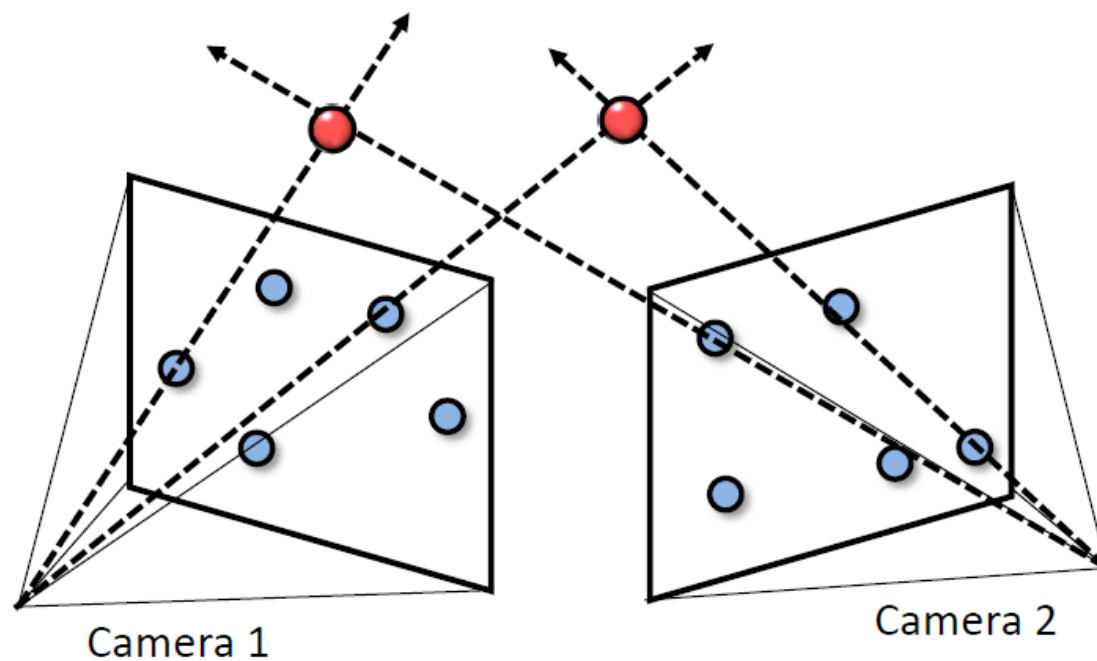


Photo Tourism — 两视图SfM

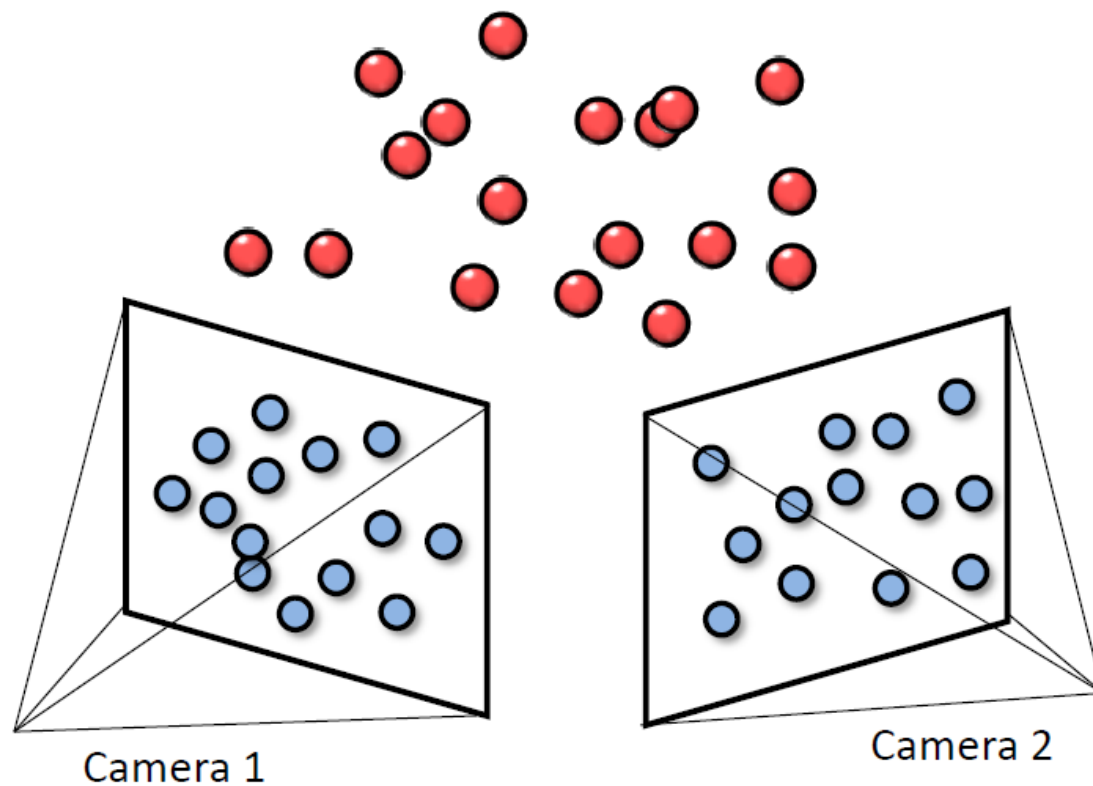
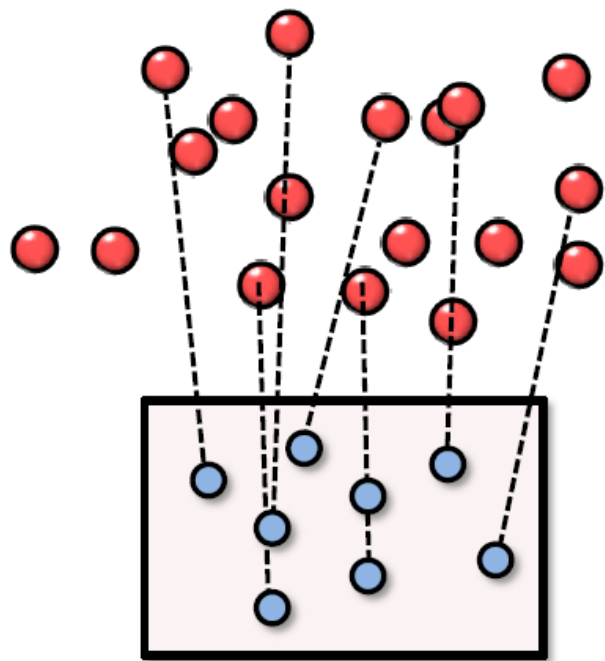
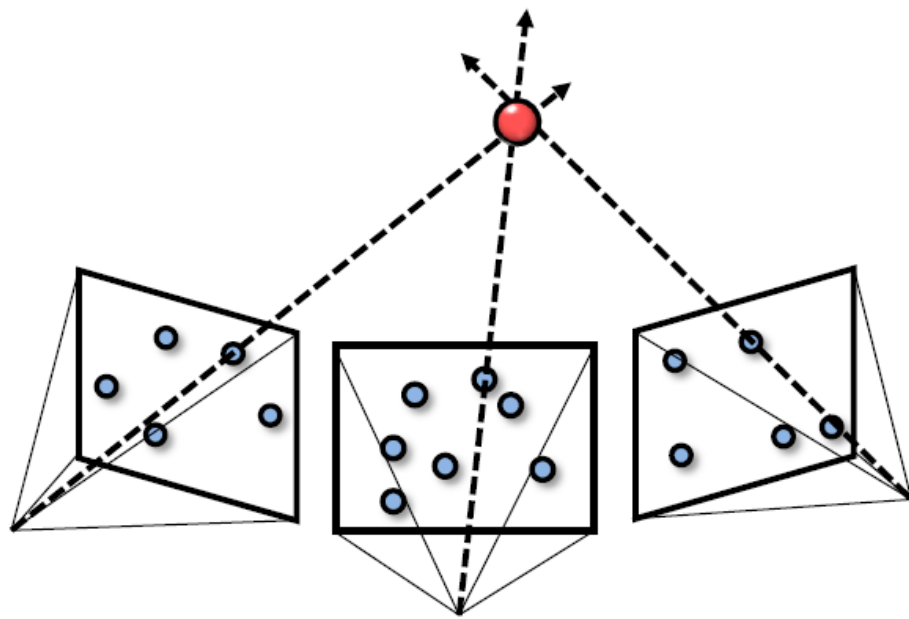


Photo Tourism — 添加新图像

- 下一步: 在初始模型中添加新图像, 同时三角化新的3D点



Pose estimation: 2D -> 3D



n -view triangulation



常用工具

• 开源工具库

- openCV: <https://github.com/opencv/opencv> 57k+ ☆
- Ceres Solver: <https://github.com/ceres-solver/ceres-solver> 2k+ ☆
- open3D: <https://github.com/intel-isl/Open3D> 5k+ ☆
- PCL: <https://github.com/PointCloudLibrary/pcl> 6k+ ☆
- CGAL: <https://github.com/CGAL/cgal> 2k+ ☆

• SfM/MVS/Meshing/Texturing开源库

- Bundler (**SfM**): https://github.com/snively/bundler_sfm 1k+ ☆
- COLMAP (**SfM+MVS**): <https://github.com/colmap/colmap> 3k+ ☆
- Theia (**SfM**): <https://github.com/sweeneychris/TheiaSfM> 600+ ☆
- openMVG (**SfM**): <https://github.com/openMVG/openMVG> 3k+ ☆
- openMVS (**MVS+Meshing+Texturing**): <https://github.com/cdcseacave/openMVS> 1k+ ☆
- MVE (**SfM+MVS+Meshing**): <https://github.com/simonfuhrmann/mve> 800+ ☆
- MVS-Texturing (**Texturing**): <https://github.com/nmoehrle/mvs-texturing> 600+ ☆



Thanks

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